Coastal Changes Predictive Modelling: A Fuzzy Set Approach

Thi Nguyen, Jim Peterson, Lee Gordon-Brown, and Peter Wheeler

***Abstract*—**Analysis of coastline variability and change is fundamental to a broad range of investigations undertaken by coastal scientists, coastal engineers, and coastal managers. Hence, various models for shoreline change modelling have been proposed. However, due to difficulties and inexactness in shoreline definitions, most models, such as linear crisp extrapolation, become unreliable for shoreline modelling. In such cases, a more reasonable and flexible modelling approach must be taken if predictive accuracy is to be improved. To this end, we present an application of fuzzy set theory in predictive modelling for shoreline changes. This approach was applied to a segment of the coast of Portland Bay, Victoria, Australia, where the erosion is effected by swell waves. The research results demonstrate the efficiency, and flexibility of the fuzzy set approach to approximation modelling with input data for the approximator being uncertain and even imprecise, as is often the case when geo-spatial data in time series is the main input data set.

***Keywords*—**Fuzzy regression, fuzzy Lagrangian interpolation, Portland Bay - Victoria - Australia, shoreline changes.

I. Introduction

COASTAL landforms respond dynamically to processes that will not always be in equilibrium. Parallel to changes in coastal erosion, sediment transport and deposition rates are consequential changes to risk management: loss of life and property, security of harbors, change of the coastal socio-economic environment, and access to coastal resources. Hence, coastal managers will be likely to call on data about rates and magnitudes of change that can be analyzed to yield predictive information.

Shorelines are recognized by the International Geographic Data Committee (IGDC) as one of the 27 most important features to be mapped and monitored (Li *et al.,* 2002). Thus some interest attaches to maintenance of information about where the shoreline is, where it has been in the past, and where it is predicted to be in the future. This is especially so among those charged with coastal management and engineering design. Various mathematical models have been proposed for shoreline change modelling. Some nonlinear methods employ complex mathematical approaches: they include some that deploy higher-order polynomial expressions (Li *et al.,* 2001), exponential expressions, or cyclic series. Nevertheless, the methods most commonly used, are linear extrapolations of a constant rate-of-change value. However, on sandy coasts, for instance, the balance between swash dominant and back-wash dominant seasons will change as will the relative significance and volumes of sand supply sources. This occurs especially on coasts responding to changes in hinterland catchment management that affect sediment transport or involve inter-regional water transfers or change in river regime (water and sediment flows) because of dam construction.

The popularity of the crisp linear approximation approach is due chiefly to its simplicity. However, it is rarely reasonable to confidently assume that a crisp function of a given form represents the relationship between the given variables. Fuzzy relations, even though less precise, seem intuitively more reasonable. Moreover, the nature of data, in some applications such as shoreline position measuring, are inherently uncertain. Accordingly, models that can better deal with uncertainties (e.g. the fuzzy sets model) need to be investigated and applied.

Fuzzy sets (Zadeh, 1965) are powerful mathematical tools for modelling and controlling such uncertain systems: they have been applied in modelling industry, human behaviour, and nature. In the absence of complete and precise information, they are decision support facilitators. As such they are of appealing value when improvements in approximation are called for. Their role is significant when applied to complex phenomena not easily described by traditional mathematics.

As early as the 1970's the modelling of fuzzy spatial events through application of fuzzy set theory was proposed, primarily in application to areas of interest in geographical analysis (Gale, 1972; Leung, 1979; Pipkin, 1978). With the development of computer-based geographic information systems (GIS), the potential of fuzzy set theory in GIS began to emerge (Robinson, 2003).

This paper aims to present an application of fuzzy set theory for coastal change modelling. The next section will examine general methodology for the approach, and the case study as well as experimental results will be stated in section 3.

I. Methodology

The general methodology in this research is shown in Fig. 1.

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In order to build the time-series shoreline database, a range of input data from a variety of sources such as aerial photos, satellite images, or paper maps could be required. Georeferencing and orthorectification procedures using ERDAS and ESRI ArcGIS software were carried out in order for all data to be projected to the same datum. Note that the shoreline herein is not a normal polyline, but it is the spatial buffer within which the shoreline will likely be found. The width of buffers depends on the quality of original sources used to extract shoreline buffers. For instance, the buffers of shorelines extracted from aerial photos are wider than those extracted from paper maps, even though the maps may have been generated from application of photogrammetric engineering.

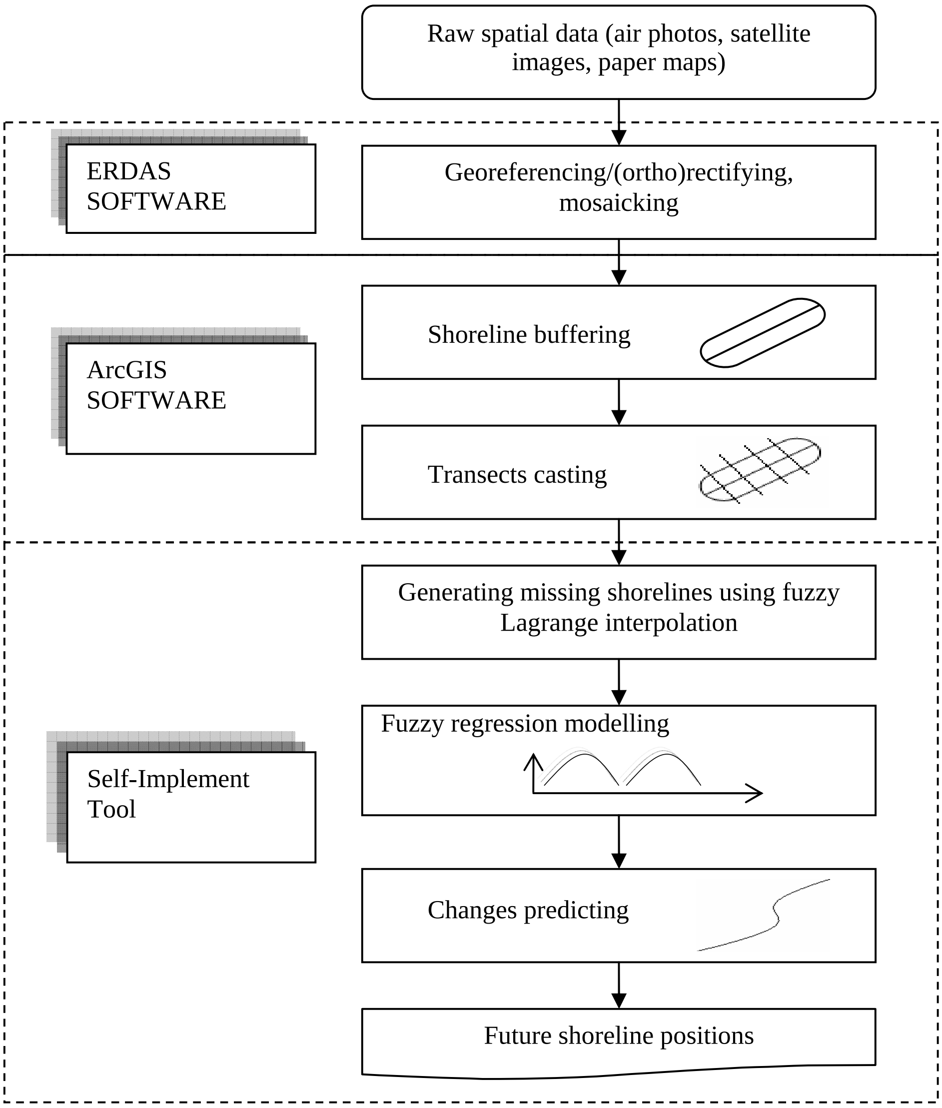

Fig. 1 Fuzzy set modelling for coastal change prediction

Transects casting (crossing the buffers) is performed using Digital Shoreline Analysis System (DSAS) version 3.x ESRI ArcGIS extension (Thieler *et al.,* 2005). This extension is implemented in VB.NET using the ArcObjects Object Library for ArcGIS 9. It was designed to aid in historic shoreline change analysis. DSAS works by generating orthogonal transects at a user-defined separation and then calculates rates-of-change and associated statistics that are reported in attribute tables (geo-spatial database). In this paper we do not focus on the use of the created geo-database to calculate rates-of-change, but via the steps described in figure 1, we use this geo-database as input data for fuzzy set modelling.

A. Generating missing shorelines using fuzzy Lagrangian interpolation

In shoreline models currently used, temporal shorelines are stored through a series of snapshots associated to particular instants in time. Therefore, information about change that occurred in the interval between two consecutive snapshots is not directly available. If the interval between the snapshots is too long, the exact beginning and duration of change that happened during this time interval is not known. Application of an interpolation method for generating missing shorelines is an essential process for overcoming this problem.

The temporal resolution equal to one year has been chosen as convenient to model the coastal changes. Due to the unavailability of existing shorelines on one year intervals, we need to generate missing shorelines at years without shorelines. After applying the annual temporal resolution, the fuzzy sets of missing shorelines would be generated based on the fuzzy Lagrange interpolation theorem and the fuzzy sets of existing shorelines.

Theorem. For $x \in R$ we have $F(x) \in \Gamma(R)$, i.e. the assignment

$F: R \to \Gamma(R): x \to F(x)$

is a well-defined function. Further we have:

(i) For all $i = 0, \ldots, n,\ F(x_i) = \mu_i$.

(ii) F is continous.

(iii) If for all i = 0, …, n, we have $\mu_i = 1_{[a_i, b_i]}$ and for each $x \in R$ we put

$$J(x) := \left\{ t \in R \mid \exists \bar{y} \in \prod_{i=0}^{n} \left[a_i, b_i\right], P^{\bar{y}}(x) = t \right\},$$

then F reduces to the function $F(x) = 1_{J(x)}$. Vector $\bar{y}$ and polynomial $P^{\bar{y}}$ of degree less than or equal to n are $\bar{y} = \left(y_0, \ldots, y_n\right) \in R^n$, and $P^{\bar{y}}\left(x_i\right) = y_i, \forall i = 0, \ldots, n$ respectively (Lowen, 1990).

Let $x_0 < \ldots < x_n$ be n + 1 points of existing years in R and let $(\mu_i)_{i=0}^{n}$ be n + 1 fuzzy sets in $\Gamma(R)$. Using the theorem, the fuzzy set representing the shoreline at missing year x will be the "fuzzy value" of function $F(x)$ defined on $J(x)$.

B. Fuzzy regression modelling

In the linear regression with fuzzy data, the dependence of an output variable on input variables is expressed by the form

$$Y = \zeta_1 X_1 + \zeta_2 X_2 + \ldots + \zeta_n X_n \qquad (1)$$

where values of input and output variables are fuzzy numbers, and $\zeta_1, \zeta_2, \ldots, \zeta_n$ are real-valued parameters.

Data are given in terms of pairs $\left\langle X^{(j)}, Y^{(j)} \right\rangle$, where $X^{(j)}$ is an n-tupe of fuzzy numbers, and $Y^{(j)}$ is a fuzzy number for each $j \in N_m$.

The aim of this regression problem is to estimate parameters $\zeta_1, \zeta_2, \ldots \zeta_n$ such that the fuzzy linear function fits the given fuzzy data as best as possible. Two criteria of goodness of fit are usually employed. The first criterion is that the total difference between the areas of the actual fuzzy number $Y^{(j)}$ and the areas of the fuzzy numbers Y_j obtained for $X^{(j)}$ by (1), should be minimised, where $j \in N_m$. According to the second criterion, the fuzzy numbers $Y^{(j)}$ and Y_j should be compatible at least to some given degree $h \in [0, 1]$; the compatibility, com, is defined by

$$com\left(Y^{(j)}, Y_j\right) = \sup_{y \in R} \min\left[Y^{(j)}(y), Y_j(y)\right]$$

The fuzzy regression can be formulated as the optimization problem:

$$\text{Minimize } \sum_{j=1}^{m} \left| \int_R Y^{(j)}(y)dy - \int_R Y_j(y)dy \right|,$$

$$\text{s.t: } \min_{j \in N_m} com\left(Y^{(j)}, Y_j\right) \geq h$$

The symmetric triangular fuzzy sets (Pedrycz, 1994) are proposed in this study. Let $X_i^{(j)} = \left\langle x_i^{(j)}, s_i^{(j)} \right\rangle$ for all $i \in N_n$ and $Y^{(j)} = \left\langle y^{(j)}, s^{(j)} \right\rangle$. Then, the fuzzy regression problem for fuzzy data becomes:

$$\text{Minimize } \sum_{j=1}^{m} \left| s^{(j)} - \sum_{i=1}^{n} \left|a_i\right| s_i^{(j)} \right|$$

$$\text{s.t. } -\sum_{i=1}^{n} \left|a_i\right| s_i^{(j)} + \sum_{i=1}^{n} a_i x_i^{(j)} \leq y^{(j)} - s^{(j)}$$

$$\text{and } \sum_{i=1}^{n} \left|a_i\right| s_i^{(j)} + \sum_{i=1}^{n} a_i x_i^{(j)} \geq y^{(j)} - s^{(j)}$$

$a_i \in R$ for all $i \in N_n$ and all $j \in N_m$

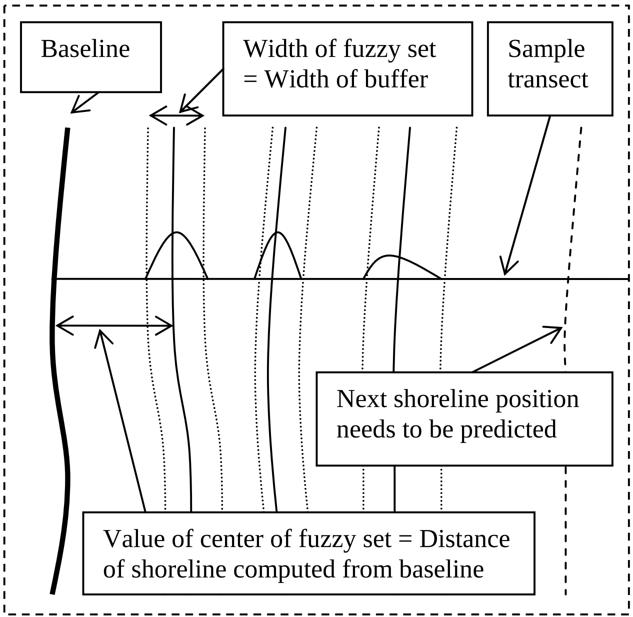

Fig. 2 Detail of fuzzy regression modelling at a sample transect (perpendicular to baseline). Three existing shorelines, for instance, are used to estimate the next shoreline position

Applying this theory in coastal change modelling, distances computed from baseline of transects crossing shoreline buffers were used as input parameters to the fuzzy regression model (Fig. 2). The regression operation is applied at each transect separately. The centres of buffers according to horizontal axis at each transect were considered as centres of fuzzy sets. The widths of fuzzy sets depend on the widths of the buffers represented.

II. Case Study and Results

A segment of approximately 2 kilometres from the east end of the basalt (rock cover) wall to Monaghan's place in Henty Bay Estate, east of Portland (Fig. 3) was used to verify the approach. At this site, the shoreline, built by sandy progradation in a swell wave environment, retreated after the building of the present Portland Harbour in 1960. The erosion is effected by waves that are extreme in magnitude. This indicates a local cause, i.e. a local realignment, and not a general one, i.e. a part of a worldwide increased erosion (Gill, 1979).

Fig. 3 Geographical position of the 2-kilometer shoreline segment

Eight shorelines available at the years of 1966, 1967, 1972, 1976, 1977, 1981, 1986, and 1992 were inputs into the model. The 1992 shoreline was used to validate the results of the fuzzy regression approach and not used in the modelling progress. There were 123 transects cast across shorelines in total (Fig. 5).

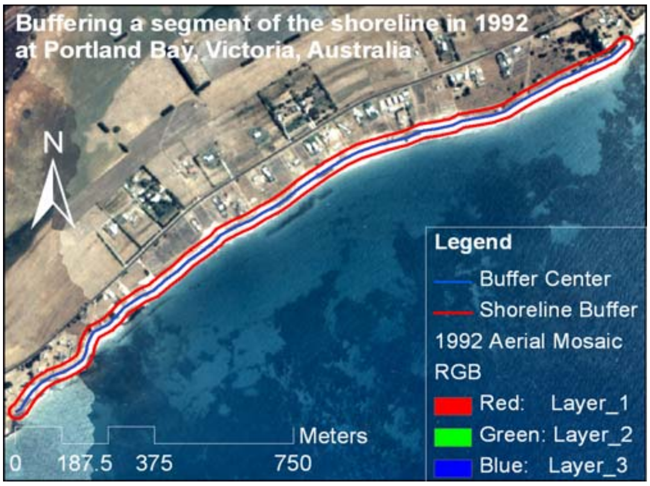

Fig. 4 Buffering the shoreline

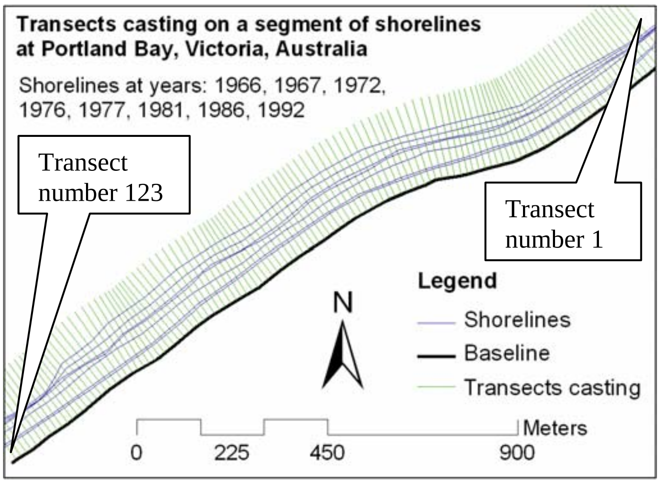

Fig. 5 123 transects casting the shorelines

To compare with the fuzzy set approach, two traditional statistics were used from among the many different methods offered by the DSAS tool: End Point Rate (ERP) and Linear Regression (LRR). These statistics are described by Thieler *et al.* (2005) as follows.

The EPR is calculated by dividing the distance of shoreline movement by the time elapsed between the earliest and latest measurements. The major advantage of the ERP is its ease of computation and minimal requirement for shoreline data. The major disadvantage is that the information about shoreline behaviour provided by additional shorelines is neglected in cases where more than two shorelines are available. Thus, changes in sign or magnitude of the shoreline movement trend, or cyclicity of behaviour may be missed.

The LRR statistics can be determined by fitting a least squares regression line to all shoreline points for a particular transect. The rate is the slope of the line. The advantages of LRR are that all shoreline data is used, the method is purely computational, it is based on accepted statistical concepts and it is easy to employ.

The result assessment of the approaches is displayed in Fig. 6. The average of absolute deviation of fuzzy set approach is 0.763 whereas that of the EPR and LRR approach is 2.861 and 2.947, respectively. It is obvious that the fuzzy set approach is much more accurate than the linear models.

Comparing results of linear and fuzzy set approach

Database sources: C:\Documents and Settings\User\Desktop\Coastal_Project\Portland_Transects_Database.xls

A	B	C	D	E	F	G	H
OBJECTID	TransectId	BaselineId	ShorelineI	Distance	Rate-Of-Change	1992 Linear Estimation (EPR & LRR)	1992 Fuzzy Estimation
1	1	1	1992	91.10240976040			91.23752832682
2	1	1	1986	80.41688076190	EPR		
3	1	1	1981	76.24157166980	-1.95	92.1168807619	
4	1	1	1977	74.05188371270			
5	1	1	1976	72.38631364260	LRR		
6	1	1	1972	70.01643826400	-1.93	91.9968807619	
7	1	1	1967	46.35719447490			
8	1	1	1966	41.42048660640			
9	2	1	1992	94.55121681490			94.344760857284
10	2	1	1986	80.83284838870	EPR		
11	2	1	1981	77.13788947320	-2.03	93.01284838870001	
12	2	1	1977	74.35868686040			
13	2	1	1976	72.56963626850	LRR		
14	2	1	1972	70.00010195800	-2.06	93.1928483887	
15	2	1	1967	43.82026670380			
16	2	1	1966	40.26635257510			
17	3	1	1992	98.08551360310			97.80980722207866
18	3	1	1986	82.91435337210	EPR		

Average of absolute deviation (|original - estimated|) of:

EPR approach: 2.861322292837398 LRR approach: 2.94688773316992 Fuzzy set approach: 0.7632147067022332

Fig. 6 Results of the shoreline change modellings. The positions of the shoreline (i.e. distance calculated from the baseline) in 1992 were estimated at all transects

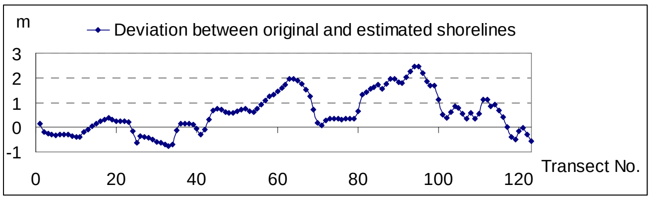

Fig. 7 Fuzzy set approach. Subtraction original distance from estimated distance at 123 transects casting the shoreline in 1992. Distances are computed from baseline

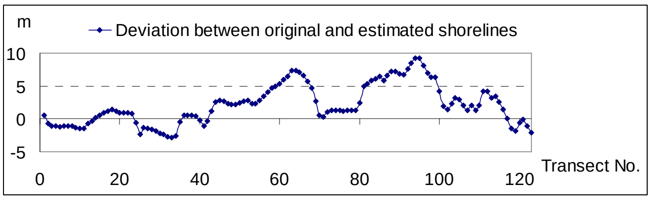

Fig. 8 EPR linear approach. Subtraction original distance from estimated distance at 123 transects casting the shoreline in 1992

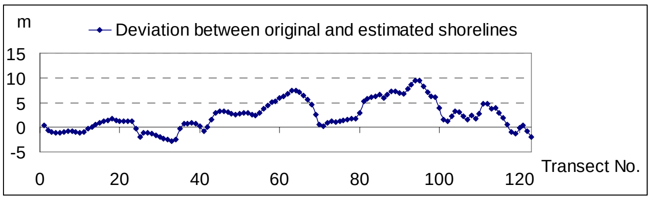

Fig. 9 LRR linear approach. Subtraction original distance from estimated distance at 123 transects casting the shoreline in 1992

The deviation between original and estimated shorelines at transects is nearly similar in terms of geographical positions when comparing among three approaches (Fig. 7-9). However, the magnitude of deviations of EPR and LRR approaches is much higher than that of the fuzzy set one. The average of absolute deviation of fuzzy set approach is much lower than that of traditional simple linear models, at the cost of a heavier modelling effort.

III. Conclusions

This paper presented an application of a fuzzy set approach for coastal change modelling. Based on the DSAS geo-database, missing shorelines were generated by using the fuzzy Lagrangian interpolation theorem and then the fuzzy regression approach is used to predict the position of the shoreline in the future. The research results demonstrated the power and reality of fuzzy sets approach in spatial change modelling, particularly when the input data is uncertain and even imprecise.

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