Earth Surface Processes and Landforms
Earth Surf. Process. Landforms **26**, 363–374 (2001)

ESTIMATING THE MEAN SPEED OF LAMINAR OVERLAND FLOW USING DYE INJECTION-UNCERTAINTY ON ROUGH SURFACES

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Received 23 March 2000; Revised 30 June 2000; Accepted 12 September 2000

ABSTRACT

A common method for estimating mean flow speeds in studies of surface runoff is to time the travel of a dye cloud across a measured flow path. Motion of the dye front reflects the surface flow speed, and a correction must be employed to derive a value for the profile mean speed, which is always lower. Whilst laminar flow conditions are widespread in the interrill zone, few data are available with which to establish the relationship linking surface and profile mean speeds, and there are virtually none for the flow range $100 < \mathrm{Re} < 500$ (Re = Reynolds number) which is studied here. In laboratory experiments on a glued sand board, mean flow speeds were estimated from both dye speeds and the volumetric flow relation $v = Q/wd$ with d measured using a computer-controlled needle gauge at 64 points. In order to simulate conditions applicable to many dryland soils, the board was also roughened with plant litter and with ceramic tiles (to simulate surface stone cover). Results demonstrate that in the range $100 < \mathrm{Re} < 500$, there is no consistent relation between surface flow speeds and the profile mean. The mean relationship is $v = 0{\cdot}56\ v_{surf}$, which departs significantly from the theoretical smooth-surface relation $v = 0{\cdot}67\ v_{surf}$, and exhibits a considerable scatter of values that show a dependence on flow depth. Given the inapplicability of any fixed conversion factor, and the dependence on flow depth, it is suggested that the use of dye timing as a method for estimating v be abandoned in favour of precision depth measurement and the use of the relation $v = Q/wd$, at least within the laminar flow range tested. Copyright © 2001 John Wiley & Sons, Ltd.

KEY WORDS: laminar flow; overland flow; dye tracing; surface runoff; flow depth

INTRODUCTION

Studies of overland flow hydraulics commonly require information on flow depths and speeds, to calculate friction coefficients or to use as independent variables in studies of soil loss. Often, these variables are calculated from the simple volumetric flow relation:

$$Q = \overline{wdv} \tag{1}$$

where Q is the volumetric flow rate (discharge) and w, d and v are the mean width, depth and speed, respectively. Given three of these values, the fourth can be estimated by substituting the known terms into Equation 1, and this makes for economy in field measurements. For example, on bounded runoff plots where flow width is predetermined or can be measured and where flow depth can be measured at sufficient test points, together with Q at the plot inlet and outlet, an estimate of the mean flow speed can be derived (e.g. Abrahams and Parsons, 1991). Measuring depth is in principle straightforward and can be done at as many points as are required, whilst flow speed is more difficult and may require a flow path of at least 20–50 cm in order to be able to time the motion of a dye cloud. In the case of flow speed, this restricts observations to 'zone' measurements rather than point observations, since there is no way to record speed variations that occur within the timed flow path. The measurement of flow speeds using dye has been refined by the division of the flow into multiple 'partial sections' within which flow conditions are relatively uniform; results from

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Contract/grant sponsor: Australian Research Council

these are then aggregated (Abrahams *et al.*, 1986). Dye speed measurements in principle allow an estimate of the mean flow depth to be derived (e.g. Katz *et al.*, 1995). Less commonly, both depth and flow speed have been determined (e.g. Dunne and Dietrich 1980).

This paper focuses on the use of dye tracing to estimate the mean flow velocity $\overline{v}$ (as required for the use of Equation 1 to estimate $\overline{d}$) in laminar flows that arise in certain temporal and spatial realms during surface runoff, and which are discussed shortly. One crucial difficulty with this approach is that the only distinct part of the dye plume that can readily be timed is the dye front (dye arrival time). Given that the fastest water flow is at the water surface, a correction factor is required in order to derive an estimate of the mean flow speed, and this factor depends upon the form of the vertical velocity profile. Very few data of good quality are available with which to determine the appropriate value for this correction factor, which is known to vary with flow state. The surface velocity considerably exceeds the profile mean in laminar flow but in turbulent flow, eddy mixing ensures that the surface flow speed exceeds the profile mean by a lesser amount.

Here, careful measurements of all four flow parameters (Q, $\overline{w}$, $\overline{d}$ and $\overline{v}$) are reported from a laboratory experiment where laminar flow was passed over a board roughened with sand grains. Thus, mean flow speeds can be estimated directly using Equation 1, without reliance on dye timing. Dye timing was used, however, and the validity of using the standard correction factor to yield $\overline{v}$ is examined. The goal is to establish whether conventional approaches to the estimation of mean flow speed in laminar flow are valid, and to evaluate appropriate values for the factor linking surface and mean flow speeds.

Velocity profiles in laminar flow

The vertical velocity distribution in laminar flow of depth d is given by the quadratic equation:

$$u = \frac{g \sin\theta}{2v} y(2d - y) \tag{2}$$

where u is the velocity, y is the distance from the bed and v is the kinematic viscosity (Roberson and Crowe, 1975). This equation applies where the surface is smooth and has a no-slip boundary condition, and where the downstream flow trajectory is unobstructed for molecules at all distances from the solid surface. For a parabolic profile where $u = 0$ at the bed and reaches a maximum (v_{surf}) at the water surface, the profile mean velocity v is reached at 0·33 of the depth, and the relationship between v and v_{surf} is:

$$\overline{v} = 0 \cdot 67 v_{surf} \tag{3}$$

Where the solid surface exhibits grain roughness, and the grains extend upward to partially obstruct flow paths, there is an ambiguity in the definition of the 'bed surface elevation' from which depths are measured. Emmett (1970) resolved this by using a blunt tip on a needle gauge, so measuring the elevation of the tops of grains roughening the bed. Others have added to the measured depth at average grain-top elevation some fraction of the grain diameter (0·25 diameters in the case of Woo and Brater 1961) to allow for the unmeasurable crevices between grains on the bed. Leaving this issue to one side for the moment, the observation remains that protruding grains obstruct flow paths, and must retard flow in the region of the grain tops. Detailed flow observations by Phelps (1975) have shown that v_{surf} is lowered relatively little (but to an extent that is a function of the relative roughness) for flow over fully submerged sand grains, and that significant flow retardation occurs within the body of the flow above the grains. As a result, the transition from retarded flow near the grain tops to maximum speed at the surface occurs over a narrower range of depths, and the vertical velocity gradient (assumed to be parabolic in Equation 3) is modified. The experimental conditions used by Phelps (1975) included only widely spaced grains whose projected cover fraction was 0·1, and there appear to be no corresponding data for surfaces completely covered by grains, like many dryland soils. Clearly, however, the presence of protruding grains alters the vertical velocity profile from the smooth parabolic form on which the relation of Equation 3 depends, and its validity in laminar flow over rough surfaces comes into question. We might anticipate that a general relationship like that of Equation

3 would apply, but take the form:

$$\bar{v} = \alpha v_{surf} \tag{4}$$

where α is a coefficient whose value may depend upon properties of the solid surface including measures of relative roughness and the spacing of the roughness elements, and the effect that these have on the shape of the velocity profile. The experiments reported here evaluate α for fully submerged grain roughness in laminar flow, such as arises on surfaces inundated by shallow overland flow, and also for flow through distributed plant litter and protruding surface stones. Both of these features commonly occur at field research sites, and are features routinely affecting surface runoff in drylands.

Prior evaluations of the value of α

A set of carefully derived data on quasi-uniform laminar flow on a laboratory sand board of fixed width was derived by Emmett (1970). He measured depth using a micrometer gauge and also surface velocity using dye-arrival timing over measured flow distances. Using Equation 1 with Q, $\overline{w}$ and $\overline{d}$ explicitly determined, Emmett solved for mean flow speed. He then compared this with surface speeds from the dye timing, and derived α from the ratio of these speeds (Equation 4). His 37 data for the range $200 < \mathrm{Re} < 2000$ (Re = Reynolds number) show the value $\alpha = 0{\cdot}576$ (std dev. 0·11: my calculation; note that all original values of Re derived in the present study, and those derived from the literature, have been brought to the common form indicated by Equation 5). Emmett (1970) noted that the value $\alpha = 0{\cdot}67$ derived for laminar flow on smooth surfaces provided only an upper envelope limit for his measured data. Emmett (1970) did not define a mean value of α for the flows exhibiting $\alpha < 0{\cdot}67$, since the value of α appeared to decline for shallower flows. However, the values lie in the range $0{\cdot}365 < \alpha < 0{\cdot}825$ (that is, some values exceeded 0·67). Certainly, α did not adopt the constant value 0·67, and indeed few values near 0·67 were recorded. Emmett speculated that extreme flow retardation arising from surface friction in very shallow flows might account for the departure from the behaviour known to apply in flow over smooth surfaces. However, even among his 'smooth' surface tests, on a board without added sand-grain roughness, values in the range $0{\cdot}37 < \alpha < 0{\cdot}59$ were found. Therefore, compared to the glass or metal surfaces commonly regarded as 'smooth', perhaps even the boards used in the tests by Emmett (1970) exhibited significant surface roughness. These data span both subcritical and supercritical laminar flow, so that the same lowering of the apparent value of α was exhibited in both regimes.

Additional experiments examining the value of α were performed by Li *et al.* (1996) and by Li and Abrahams (1997), for both clear water and water carrying saltating sediment grains. Once again, it was found that α did not exhibit a value of 0·67 for laminar flow, nor did it adopt the theoretical value of 0·8 for turbulent flow. Li and Abrahams (1997) found that α was lowest in laminar flow (where no distinct trend with Re could be shown), rose steeply through the transitional range of Re, and continued rising slowly in the turbulent range. Interestingly, for the assumed laminar range ($\mathrm{Re} < 2000$), Li and Abrahams (1997) found the mean value of α to be 0·37, which is much lower than the values reported by Emmett (1970). The median sand grain size contributing surface roughness in the tests of Li and Abrahams (1997) was 0·74 mm, whilst Emmett (1970) used sand grains of 0·5 mm median diameter. This raises the possibility that the value of α declines as surface grain roughness increases. To my knowledge, this possibility has not been systematically investigated. However, if it is real, then such a marked dependence of α on grain size poses severe difficulties for the use of dye tracing to estimate flow speeds on natural surfaces where grain roughness height is markedly variable.

A further uncertainty that remains is how α behaves in the lower range of laminar flows, far from the region of transition to turbulence. The present study attempts to fill that gap by concentrating on the range $100 < \mathrm{Re} < 500$. Though such flow Reynolds numbers are common in overland flow on gently sloping dryland surfaces (Dunkerley, unpublished data), no studies of flows in this range, apart from a few observations by Emmett (1970), have been published. To put this range of Re in perspective, on a strip of sloping terrain 1 m in width, and taking a representative mean overland flow speed of 5 cm s^{-1}, if rainfall intensity is 30 mm h^{-1} and soil water uptake is 15 mm h^{-1} (reasonable values for drylands of western New South Wales, Australia) (Dunkerley, in press), a slope length of 30 m would be required to reach Re = 500; a runoff fetch of 6 m is needed even to reach Re = 100. Thus, over significant parts of the interrill realm, flows will have Re values < 500. On the assumption that the threshold value of Re marking the beginning of turbulent flow was 1200, Woolhiser *et al*. (1970) concluded from rangeland experiments in South Dakota that laminar flows persisted for flow lengths of up to 170 m. Thus, it is important that the behaviour of α in these widespread flows be examined. Conditions in the early stages of overland flow would be affected by the impact of raindrops. Variation in the magnitude of this disturbance would vary with flow depths, and with drop sizes and rainfall intensities. These further potential effects on the value of α are not considered here.

Despite the early findings of Emmett (1970), and later Li *et al*. (1996) and Li and Abrahams (1997), there has been widespread use of the value $\alpha = 0{\cdot}67$ in the process of estimating mean flow speed from surface values for flows in the laminar range. For example, this was done in the field for flows that included laminar regime flows over gravel-covered surfaces by Abrahams *et al*. (1986) who used dye speeds measured over flow lengths of 25 cm. The value $\alpha = 0{\cdot}67$ was also adopted for the conversion of surface velocities in the laboratory flume tests of Guy *et al*. (1990), where flows were passing over a sandy test soil, by Rouhipour *et al*. (1999) for field trials in a pineapple farm where water was trickled onto the top of test furrows, and by Fox and Bryan (1999) in a laboratory study of interrill erosion in soil boxes exposed to artificial rain. Similarly, for turbulent flows, the corresponding theoretical value $\alpha = 0{\cdot}8$ has been systematically adopted (e.g. by Nearing *et al*. (1999) in a study of rill flow hydraulics). Dye timing has also been used in a range of flow roughness studies, in some cases (e.g. Roels 1984) without any conversion to profile mean speed. In none of these cases where a value for the coefficient α was adopted was evidence presented that the value had been justified except by extrapolation from the known behaviour of viscous (or turbulent), sediment-free flows on smooth surfaces.

To my knowledge, there are no studies systematically investigating the effects of protruding roughness elements, or organic litter, on the value of α applicable to laminar flows. If dye tracing is to be used to estimate mean flow speeds on natural plots, it must be established whether or not there is an appropriate value for α for use when the effects of protruding obstacles and litter are present.

MATERIALS AND METHODS

Experiments were conducted on glued-sand boards of $0{\cdot}6 \times 1{\cdot}2$ m, made by applying a heavy coat of waterproof varnish and sprinkling this with sand while still wet. The excess sand was then removed. By sieving and calculation of the mass-weighted mean diameter, the mean grain size of the two sands used was found to be 0·4 mm for medium sand board and 0·67 mm for coarse sand board. The coarse sand was less well sorted than the medium sand. Side walls 2 cm in height, and roughened with sand in the same way, were attached to each side of the flow boards, to make a confined channel 0·5 m wide and 1·2 m long. Each board was carefully levelled laterally and inclined at a slope of 1·2 ° by the use of wedges. This gradient typifies the slope of runoff–runon landscapes in arid Australia (e.g. Dunkerley and Brown (1995) studied the banded vegetation of western New South Wales, Australia, that has developed on slope gradients lying in the range 0·5–2·1 °).

Water was recirculated from a tank using a variable-speed pump, and fed to the top of the board through a perforated pipe having eight outlet holes spaced evenly across the width of the board. Unit flow rates were varied in the range 0·25 to 1·14 cm^2 s^{-1}; these were associated with flow depths ranging from about 0·8 to 2·7 mm. Flow passing from the bottom of the board was caught in a gutter, at whose outlet pipe flow rate was measured volumetrically every 2 min during tests which lasted about 15 min. Water temperatures were recorded at the same intervals using a platinum resistance thermometer and recorded to the nearest 0·1 °C, and kinematic viscosity calculated using the polynomial relationships of Weast (1979).

For each test condition and flow rate, drops of fluorescein dye were added to the flow and the surface flow speeds recorded from travel time of the dye at three locations across a test section of 50 cm mid-way along the board. In an attempt to minimize errors arising from the reaction time of the observer, anticipation was used in judging when to operate the stopwatch controls. That is, in releasing dye, an attempt was made to depress the syringe and the stopwatch 'start' button simultaneously. Likewise, the dye front was closely observed as it approached the lower boundary of the flow path marked out for timing so that the watch could be stopped simultaneously with the arrival of the dye front. When a mistake was made (beginning or ending timing too soon or too late) the dye test was discarded and the test repeated.

In each of the surface treatments, described below, four to six different flow rates were applied to the boards.

All surface treatments were applied to the medium sand board. One set involved placing obstacles on the sand board to simulate surface stones protruding through the shallow overland flow. In order to avoid the complication of irregular obstacles that might not touch the surface except at their extremities, and for which areal cover fraction would thus be of limited use (e.g. irregular natural pebbles), glazed ceramic tiles of two sizes were used. These had surfaces of the same smoothness as is typically found for the stones littering the desert surface, which are mostly of glassy vein quartz or other resistant materials carrying a desert varnish. The ceramic tiles were set out randomly on the board to yield cover fractions of 5, 10 and 20 per cent. The tiles used had a thickness of 6 mm, sufficient to ensure that they were never overtopped by the flow, and remained as obstacles which generated a larger upslope wetted area as flow depths were increased. Tiles were all placed with their upslope face normal to the flow, so as to present the most blunt obstacle possible.

The other treatment involved the application of plant litter collected from the Broken Hill region of arid western New South Wales. Three loadings were used (20, 40 and 80 g m^{-2}) with the litter sprinkled uniformly over the board. These loadings lie within the ranges found by various workers in dry shrublands and grasslands, where litter cover can reach 20–70 per cent (Woolhiser *et al*., 1970; Johnson and Gordon, 1988). As measured by grid counting, these litter loadings provided approximately the same areal cover fractions (5, 10 and 20 per cent) as employed in the ceramic tile experiments. Except for some of the smallest litter fragments, which moved episodically with the flow, the litter was immobile in the range of flows used.

To allow flow to reach equilibrium speed down the board, and to avoid edge effects at the downslope lip (where surface tension effects cause water to pond before spilling over), all depth measurements were made in a central test area of the board located at the mid-point of its length, and measuring 50 cm $\times$ 50 cm. Within this area, depths were measured on a grid of 64 points, using a computer-controlled gantry that was levelled above the sloping board. Stepper motors were used to move a measuring carriage to nominated (X, Y) coordinates on this grid, and at each point an electronic needle-gauge was lowered by a precision stepping system until a circuit was closed when the needle tip contacted the water surface. This Z probe had a resolution of 0·025 mm (<0·001 inch). To map elevations across the dry board, the same Z probe carried an opto-electronic switch that was activated when a solid object was touched. This had the same 0·025 mm resolution, and the needle point probe had a diameter of 0·35 mm (smaller than the diameter of the sand grains) and was additionally drawn to a narrow pointed tip. Water depths at each grid point were then derived by subtracting the elevation of the dry board at that point from the elevation of the water surface at the same grid point once flow had been turned on. The X–Y system returned to each grid point within a tolerance of 0·25 mm. Data were logged directly onto a laptop computer linked to the motor control electronics, and the time and coordinates of each point recorded. In addition, the data file recorded whether the needle probe had touched the water surface or whether the optical switch had been activated by a solid object. In this way, it was possible to decode from the file those points where a roughness element lay beneath the grid point. These non-submerged points were excluded when mean depths were calculated (Abrahams and Parsons, 1990).

Calculation of flow parameters

Flow Reynolds numbers were calculated from the relation:

$$\mathrm{Re} = \frac{4\bar{v}\bar{d}}{\bar{\nu}} \tag{5}$$

and the Froude number was determined from:

$$F = \frac{\bar{v}}{\sqrt{g\bar{d}}} \tag{6}$$

To characterize retardation of flow, the Darcy–Weisbach relation

$$f = \frac{8g\bar{d}S}{\bar{v}^2} \tag{7}$$

was employed, with S being the board slope. The four to six different discharge rates employed allowed the same number of values of f to be derived. These were then used to derived a roughness value, K, for the surface treatment, using the relation:

$$f = \frac{K}{\text{Re}} \tag{8}$$

RESULTS

Dye tracings confirmed laminar flow on the bare sand boards, with no mixing of adjacent flow filaments. Dye paths likewise showed that flow was diverted in smoothly curving paths to pass between and around the ceramic tiles, which rested flat on the board surface so that no flow passed beneath them. Areas of stalled flow upslope, and areas of flow separation downslope, were noted to arise from the square tiles placed with leading edge normal to the flow. Flow was not steady, but accelerated through constrictions between neighbouring tiles, and slowed where less of the board width was occupied by these obstructions. Similarly, even in the presence of the highest litter loadings, flow paths down the board were smoothly curved and of low sinuosity. Owing to the angular shapes of twigs, thorns and flower parts, flow was able to pass beneath litter particles, or through gaps in clumped litter. Absence of mixing of adjacent filaments again indicated that laminar flow conditions were maintained at all discharges.

The numerical description of laminar flows on the medium sand board is considered first, since this board was studied in greater detail than the coarse sand board, and was used for all subsequent tile and litter treatments. The five discharge rates used resulted in $103{\cdot}8 < \text{Re} < 410{\cdot}2$ (Table I), while all flows were subcritical. Mean depths directly measured by the probe system lay in the range 0·85–1·22 mm, while mean flow speeds calculated from Equation 1 using these depths lay in the range 2·98–8·23 cm s^{-1}. Surface flow speeds were $5{\cdot}28 < v_{surf} < 14{\cdot}19$ cm s^{-1}. The Darcy–Weisbach f was 1·56 at the lowest flow rate, and declined to 0·29 at the highest. Overall, the value of K for this board (Equation 7) was 138. From lowest to highest discharge, values of α lay in the narrow range 0·56–0·61, showing no tendency to change systematically with the value of Re.

On the coarse sand board, results were similar in all respects, though flow speeds were lower, and the derived value of K (Equation 8) for this board was higher, at 199, as expected in view of the coarser sand grain size.

The litter and tile treatments resulted in complex patterns of change in depth, velocity, Darcy–Weisbach roughness, and other flow parameters (Table I). These are not pursued in detail here, in order to focus on the behaviour of α. For large tile tests, $0{\cdot}53 < \alpha < 0{\cdot}64$ (mean 0·561), and for small tile tests $0{\cdot}47 < \alpha < 0{\cdot}69$ (mean 0·574). In the litter tests, $0{\cdot}50 < \alpha < 0{\cdot}61$ (mean 0·543). In comparison, the mean from both bare sand boards was $\alpha = 0{\cdot}579$. There is no statistical difference among these populations at $p \leq 0{\cdot}01$ (small-sample t-test; Freund 1974). In view of this, the results can be pooled and across the tested range $100 < \text{Re} < 500$ this yields a mean value of $\alpha = 0{\cdot}56$.

There is no relationship between α and Re in the present data. However, in the plot of α versus Re (Figure 1) it is notable that the lowest values of α seem to be associated with the heaviest litter loading and with the 10

Table I. Ranges of key test conditions and resulting flow properties for bare sand board, ceramic tile and plant litter tests using laminar flow

Board treatment	K (Eqn 8)	Range of Q tested ($cm^3 s^{-1}$) and (number of increments of Q)	Range of Re (Eqn 5)	Range of $\bar{d}$ (mm)	Range of $\bar{v}$ ($cm s^{-1}$)	Range of α (Eqn 4)	Mean error in using $\bar{v} = 0{\cdot}67\ v_{surf}$ (%)
Bare medium sand	138	12·6–50·1 (5)	103–410	0·85–1·22	2·98–8·23	0·56–0·59	14·4
Bare coarse sand	199	22·5–56·4 (4)	190–485	1·11–1·74	4·05–6·67	0·43–0·58	26·5
5% cover small tiles	142	16·0–53·5 (5)	137–465	0·91–1·33	3·69–8·41	.63–.69	1·35
10% cover small tiles	233	13·3–53·6 (5)	119–478	0·98–1·68	3·01–7·08	0·47–0·57	30·9
20% cover small tiles	211	13·9–48·3 (4)	141–490	1·00–1·64	3·47–7·34	0·55–0·61	15·8
5% cover large tiles	171	12·3–49·3 (5)	106–423	0·88–1·35	2·91–7·69	0·54–0·59	20·4
10% cover large tiles	133	12·7–50·8 (5)	107–429	0·84–1·33	3·37–8·48	0·53–0·64	16·5
20% cover large tiles	177	14·8–40·0 (4)	141–380	1·02–1·38	3·62–7·25	0·54–0·62	17·3
20 g m^{-2} plant litter	264	12·9–47·2 (4)	106–385	0·96–1·59	2·68–5·93	0·54–0·61	17·7
40 g m^{-2} plant litter	288	13·4–39·8 (4)	108–319	1·06–1·54	2·53–5·18	0·54–0·61	18·3
80 g m^{-2} plant litter	1015	13·3–53·3 (6)	103–410	1·51–2·67	1·76–4·00	0·50–0·59	26·5

per cent cover of small tiles; in both cases these treatments resulted in high values of the Darcy–Weisbach f. The highest values of α arose from some of the large tile tests, and these were associated with low values of the Darcy–Weisbach f. The bare sand board flows lie in about the middle of this range. These observations suggest, rather tenuously, that there may be an inverse relation between α and the Darcy–Weisbach f in the

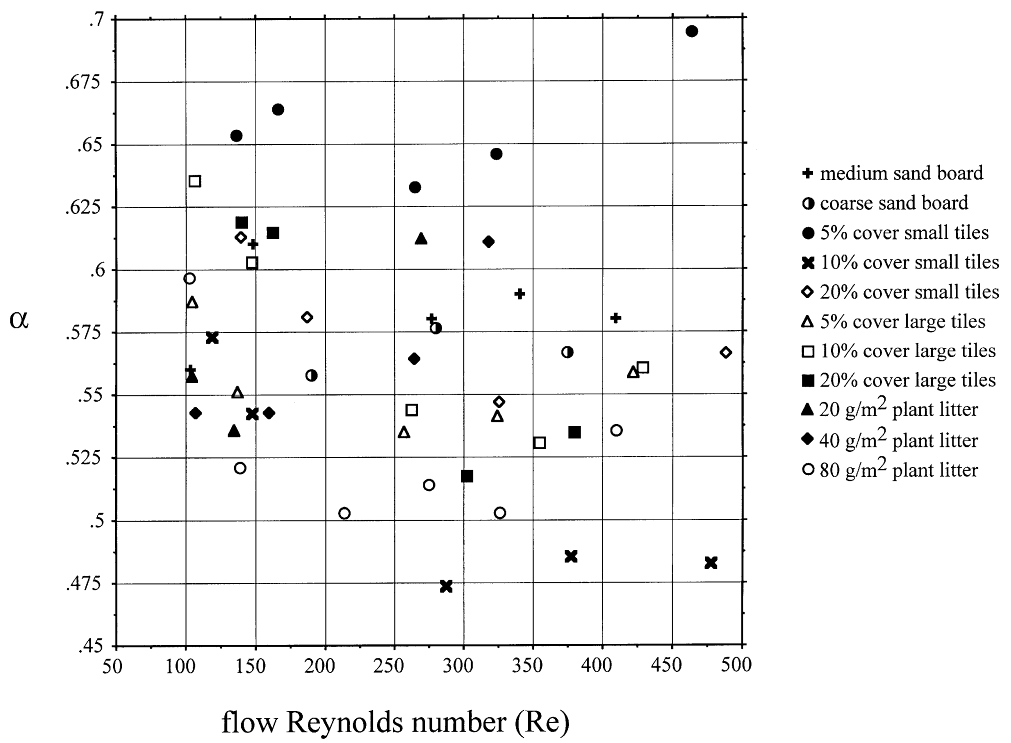

Figure 1. Relationship between the ratio of surface and mean flow speeds, α (Equation 4), and the mean flow Reynolds number for the laminar flow experiments

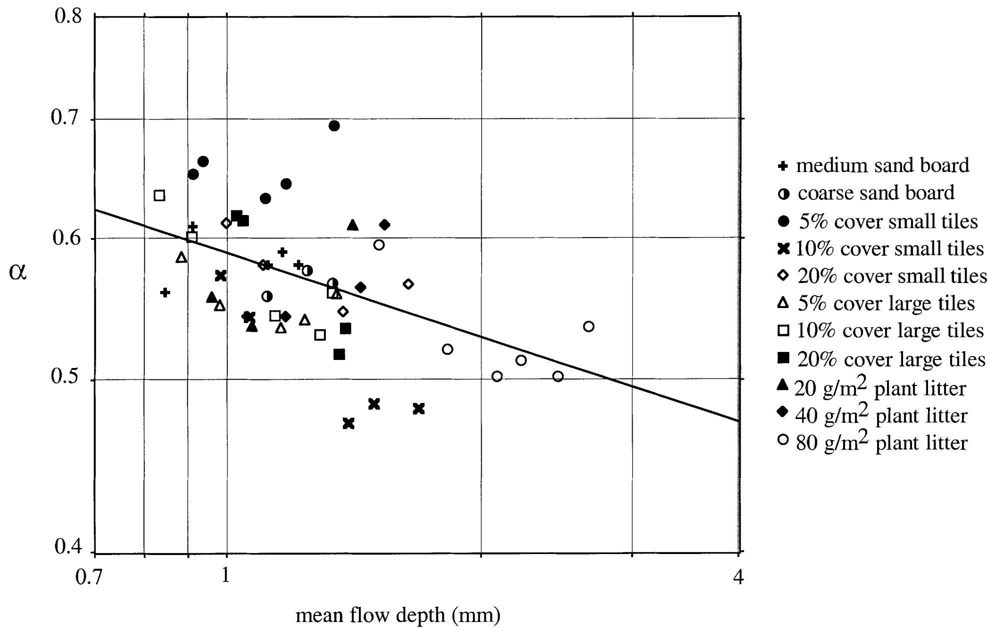

Figure 2. Relationship between the ratio of surface and mean flow speeds, α (Equation 4), and the mean flow depth for the laminar flow experiments. The solid line represents the least-squares power function regression model fitted to these data

range 100 < Re < 500. Statistical tests of significance do not support this contention, but it is noted as a hypothesis that may be worthy of further testing.

A statistically significant relationship was found between α and mean flow depth (Figure 2). This relation was best fitted as a power function taking the form:

$$\alpha = 0 \cdot 587\overline{d}^{-0 \cdot 165} \tag{9}$$

For this relation, $r^2 = 0{\cdot}27$ and $p = 0{\cdot}0001$. This indicates that there is a significant decline in the value of α in deeper flows, regardless of the treatment (bare sand board, small or large tile cover, or litter loading) although the scatter in the relationship remains large. In very shallow flows, the value of α approaches most closely the theoretical smooth-surface value of 0·67, and the value falls as flow depth rises. This relationship is concealed when using Re as the independent variable, suggesting that changes in depth and flow speed to some extent offset each other in ways that require further study.

DISCUSSION

In all of the tests made, the adoption of $\alpha = 0{\cdot}67$ to estimate mean flow speeds from surface dye arrival times results in overestimates of v, the mean difference being 18·7 per cent and the maximum nearly 31 per cent. This is a sizeable error, but possible measurement problems that might arise in observing thin flows must be eliminated as possible causes.

The observation of v_{surf} using the stopwatch is the first potential source of difficulty. It is unclear just what errors of judgement remained in the stopwatch timing despite the careful use of anticipation noted earlier. It seems safe to deduce that the use of anticipation would reduce the normal reaction time that would apply in responding to an event occurring without warning, which is commonly a few hundred milliseconds (Welford

1980). If, therefore, the travel time of the dye front was misjudged by say 200 ms (half of a plausible 400 ms arising partly at the moment of dye release and partly at dye front arrival downslope), then the error amounts to <5 per cent for slower, shallow flows, but up to 13 per cent for the deepest and fastest flows. However, several repeat timings were made for each flow condition, and the mean travel time taken. Judgement errors should involve both erroneously short and long travel times, so that resulting errors in the mean should be less than those of the individual dye observations. Furthermore, it is known that practice greatly reduces the reaction time (Welford 1980), and anticipatory stopwatch timing was used repeatedly and under identical conditions throughout the experiments. In order to account for a 30 per cent speed error in a test where the travel time of the dye front was near 6 s (a common value), the timing error would have to be nearly 2 s, which does not appear likely, though timing errors of 13 per cent would nevertheless be of concern in the very fastest flows. Evidence against a major role for timing errors in even these flows is, however, provided by an examination of the relationship between surface dye speeds and the discharge. In all cases the relationship was extremely strong across all observations from lowest to highest discharge (coefficients of determination in the regression relation between v_{surf} and Q were in all cases >0·99). Had increasing haphazard timing errors been affecting the dye timing, these relationships would be expected to show increasing scatter at higher values of Q, and this was not observed. Therefore, it does not seem probable for timing errors to be the cause of the 15–30 per cent difference between the speeds based on dye timing and those calculated using observed flow depths together with Equation 1, and indeed, given the very tight regression models linking v_{surf} and Q, it would appear that substantial errors arising from timing are not present in the data.

Consideration must now be given to the potential errors in the instrumental observation of flow widths, depths and flow rates.

Flow width was fixed at 50 cm in all tests and cannot be in error, at least for the bare sand boards. For the tile experiments, obstacle size was uniform and the appropriate reduction in flow width to allow for the protruding obstacles was applied. In the case of litter, there is much more ambiguity about how flow width ought to be determined. In view of the observation that no flow path along the board was totally obstructed by litter (the flow passing above, under or through litter particles and litter clumps), width was in all cases taken to be 50 cm. Likewise, Q was timed repeatedly using a stopwatch and graduated laboratory vessels, and across the several hundred measurements reported cannot consistently have been underestimated by nearly 19 per cent as would be required to account for the discrepancy between v and v_{surf}. Rather, it would be expected that errors would be small, and about half too low and half too high.

This leaves only depth measurement error, and this is perhaps the most problematic of the variables to observe with extreme precision. However, depth would have to be consistently overestimated by about 23 per cent to account for the discrepancy in flow speed calculations. In fact, the probe mechanism used to measure depth is more likely to underestimate flow depth, rather than overestimate, for two reasons. Not even the narrow tip of the needle probe can pass into the small depressions between closely spaced sand grains, so that the apparent elevation of the roughened surface would be too high. But the error here cannot exceed about 0·5 grain diameters, since the grains are packed closely together and would touch at about 0·5 grain diameters above the varnished surface of the board. Further, this error is compounded because of the mode of operation of the probe system. The stepper mechanism moves the needle probe in steps of fixed size (0·025 mm). This incremental stepping mechanism results in worst-case errors when a downward step brings the probe tip very near to the water surface. The next downward step would then carry the tip below the surface (i.e. an overshoot), since the smallest downward step of the probe is 0·025 mm. Given that water depth is determined from the distance between the downward travel distance to the bare board and the downward travel to the water surface, the slight overshoot possible in this worst-case analysis tends to result in depths that are too low. Given this maximum error of 0·025 mm, the largest depth error arising from needle probe overshoot for flows 1 mm deep is an underestimate of 2·5 per cent, and 1·25 per cent for flows 2 mm deep. However, consistently worst-case errors could not arise systematically, and most depth measurements would certainly have involved less error. If the inability of the probe to sense tiny gaps between the sand grains is added, and estimating this to have a mean of 0·2 grain diameters, the total underestimate of depth on the medium sand board could amount to 0·105 mm, or 10·5 per cent in flows 1 mm deep, and 5·25 per cent for 2 mm flows. But these are underestimates, and overestimates of depth are required to account for the speed discrepancy. Therefore, it seems almost impossible for depth errors to be responsible for the results.

Finally, the discrepancy in speed calculations could be accounted for by a combination of consistent errors, such as measurements of Q consistently too low by nearly 10 per cent and measurements of depth consistently too high by 10 per cent, but this again seems improbable through more than 3000 depth observations taken during the study, and several hundred careful measurements of Q. The conclusion is that the discrepancy in flow speed estimates is real and not a measurement artifact.

Having ruled out artifacts of the measurement system, we can observe that the mean value of $\alpha = 0{\cdot}56$ closely resembles the value derived earlier from Emmett's (1970) data, of $\alpha = 0{\cdot}576$. Emmett's values span several different slope angles and a range of rough and smooth surfaces, but with roughness elements all fully submerged. These conditions are identical to those used in the present tests on the bare sand boards, but are distinct from both the tile and litter tests. Li and Abrahams (1997) also found values of α to lie below 0·67 in the laminar range, but their result was somewhat lower, with a mean of 0·37. Li and Abrahams (1997) found no clear support for any relation between α and Re for laminar flow, and this is consistent with the present results.

The results confirm the findings of earlier workers, cited previously, that the behaviour known from viscous 'smooth flow' cannot be transferred to rough artificial or natural surfaces without incurring error. The mismatch between mean flow velocity estimated using Equation 1 and converted surface velocities estimated from dye timing reaches a maximum of 30·9 per cent (Table I). Whilst this is a substantial error, when the resulting erroneous values are carried into subsequent calculations, the errors can be compounded. For example, when the resulting estimates of mean velocity are used to estimate the Darcy–Weisbach roughness coefficient f (Equation 7) (e.g. Dunne and Dietrich 1980), the errors are increased because a term in $\bar{v}^2$ appears in the denominator. Thus, a velocity in error by 20 per cent produces an error in $\bar{v}^2$ of more than 30 per cent, and an equal underestimate of the surface roughness. In contrast to the moderate error in flow speed estimation, this error in roughness coefficient can become severe, and could swamp detection of roughness variability arising from different surfaces under investigation, leading to a failure to achieve experimental objectives.

The values of α derived here are notably higher than those derived by Li and Abrahams (1997), who found values approaching 0·3 for Re = 500 (the lowest Re value examined by them). The data of Li and Abrahams (1997, figure 2) show α rising steadily from this minimum in the laminar region, and through the transition to turbulence. It seems unlikely that the curve rises steeply once more at lower values of Re to reach a mean of 0·56 as found here in the range $100 < \mathrm{Re} < 500$. Therefore, it is difficult to reconcile the present results with those of Li and Abrahams (1997). It is worth emphasizing that the coarse sand board used here has about the same mean grain size as that employed by Li and Abrahams (1997), whilst the litter treatments tested provided a test condition with a dramatically higher Darcy–Weisbach f. Despite the fact that the ranges of Re tested in these two studies do not overlap, they nonetheless abut one another, and the difference in α values warrants closer inspection.

Li and Abrahams (1997) found from trials on a glass surface that α was lower on this smoother surface than on their sand board. They also found that their use of concentrated salt as a tracer in measuring flow speeds was associated with underestimation of α, possibly because the dense saline solution travelled at the bottom of the water column where flow speeds are lower. On glass at Re = 1056, the value $\alpha = 0{\cdot}30$ derived from salt tracing was less than half of the value ($\alpha = 0{\cdot}61$) derived through the use of a point gauge and solving for v in Equation 1. This is a considerable discrepancy, and it is notable that the value derived by needle-gauge measurement of depth is very similar to the values derived in the present study.

Whilst in the present study no good relation was found linking α and Re, a very strong one was established linking α and mean flow depth. Given the formula for Re (Equation 5), and the negligible variation in kinematic viscosity during the experiments, this is informative. It suggests that flow depth is involved physically in setting the value of α, but that this is concealed when $\bar{d}$ is entered as a term in the formula for Re. Thus, co-variation in $\bar{v}$ must nullify the effect of changing depth, so that their product remains relatively unaltered. For example, while flow depth increases in the presence of protruding obstacles in the flow, velocity is not sensitively affected. Dunkerley *et al*. (unpublished data) showed that for obstacles protruding through laminar flow, mean flow speeds can either be increased or decreased, depending on the geometric arrangement of the obstacles. Narrowing of the flow paths offsets the obstacle drag because in the laminar range, the deeper resulting flow paths are associated with higher values of Re and therefore lower overall flow resistance. In the presence of litter, flow depths are notably increased, but correspondingly, flow speeds are reduced. Thus, again, the value of Re can change less than the values of the depth and speed terms used to derive it (Dunkerley *et al*., in press).

CONCLUSIONS

In view of the scatter of values for α derived here, it is suggested that estimation of mean flow speeds by the conversion of leading-edge dye arrival times does not provide a suitable basis for research investigations, at least within the range $100 < \mathrm{Re} < 500$, which encompasses the range of at least some overland flows (see earlier discussion). Natural soil surfaces, in particular, have a wider range of roughness elements than laboratory sand boards, and may have surface stones, litter, faunal mounds and other kinds of microtopography. All of these will perturb the flow and the vertical velocity profile, and so modify the value of α. It does not appear feasible for a worker undertaking an experimental programme where data on runoff speeds are needed to be able to characterize both the surface roughness and other features in great detail, merely to estimate v. Instead, it is the writer's recommendation that either direct velocity determinations be made (e.g. using laser doppler or hotwire methods) or that flow depths be observed using careful needle-gauge observations. Failing this, it appears that data of doubtful value will be generated when adopting any value for α whose applicability to the particular test conditions has not been demonstrated.

An important issue not addressed in the present work is what property of the rough surface over which the viscous flow passes is responsible for the setting of the appropriate value of α. It seems likely that this is the height and density of the roughness elements that protrude significantly into the body of these shallow flows, or through them in the case of litter clumps or surface stones. However, this is an area where further investigation is required.

ACKNOWLEDGEMENTS

I gratefully acknowledge the skilled technical help of Peter Domelow (Department of Biological Sciences) and David Tooth (School of Geography and Environmental Science) at Monash University in the design and construction of the measuring gantry used in this study. Thanks are also owed to the two reviewers of the manuscript for their very valuable suggestions for improvement. The work was supported by a grant from the Australian Research Council.

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