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VOLUMETRIC DISPLACEMENT OF FLOW DEPTH BY OBSTACLES, AND THE DETERMINATION OF FRICTION FACTORS IN SHALLOW OVERLAND FLOWS

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ABSTRACT

Large roughness elements such as stones or plant stems (*obstacles*) influence the depth of overland flows in two ways. The first effect is a dynamic one, involving frictional retardation of the flow and associated reduction in flow speeds. The second influence is static, and arises from the upward volumetric displacement of flow depth because of the submerged volume of the obstacles. Depending upon the distribution of submerged obstacle volume with height above the soil surface, the proportion of the flow volume occupied (and so, the perturbation of flow depth arising from volumetric displacement) can vary irregularly or systematically with flow stage. Furthermore, the amount of volumetric displacement of flow depth would vary among surfaces carrying different cover fractions of identical obstacles. Consequently, estimates of the change in friction factors arising from the drag on flow traversing varying obstacle cover fractions are confounded with the parallel shift volumetric displacement. To understand the true frictional drag arising from obstacles, a correction must be made for the volumetric displacement. A method for making this correction is outlined. New laboratory experiments provide precise observations of depths and friction coefficients in laminar flows passing fields of regular obstacles. After making the proposed correction for volumetric displacement, increases of 40 to 75 per cent in the derived value of the Darcy–Weisbach friction factor, f, are found for an obstacle cover of 20 per cent. Many published studies of friction coefficients in shallow overland flows, such as those on stone-covered dryland soils, involve larger obstacle cover fractions, and evidently involve the significant confounding effect of volumetric displacement. Copyright © 2002 John Wiley & Sons, Ltd.

KEY WORDS: laminar flow; friction factor; flow depth; volumetric displacement

INTRODUCTION

An understanding of the hydraulics of shallow overland flows, both laminar and turbulent, is required for modelling the initial stages of runoff and soil erosion. One important research goal is to understand the ways in which frictional retardation is generated by varying roughness elements that include the mineral grains forming the soil surface, and larger elements such as stones, litter and standing plant stems (here referred to as *obstacles*) that lie on or above the soil surface. This understanding is a precursor to successful modelling of the hydraulics of surface runoff, and of sediment removal in such flows, as tools for land management. Considerable effort has been invested in approaching this goal through laboratory and field experimentation (e.g. Gilley *et al*., 1990, 1992; Abrahams and Parsons, 1994;, and other references cited later). The goal here is to explore a neglected aspect of the study of flow through obstacles, namely, the effect on flow depths (and hence on calculated friction coefficients: see Equation 1 below) of the volumetric displacement arising from the submerged parts of the obstacles. This effect arises even in stationary, ponded water, and the depth changes that it produces are separate from those related to the frictional retardation of flowing water.

Flow depth provides a key term in the evaluation of the frictional drag on a surface carrying overland flow, and one that is readily measured in the field. For example, the Darcy-Weisbach friction factor f, which can be applied to both laminar and turbulent flows, is widely used in the analysis of flow retardation, and is

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estimated from observations on flow depth, slope and velocity, using the relationship

$$f = \frac{8g\overline{d}S}{\bar{v}^2} \tag{1}$$

in which $\overline{d}$ is the mean flow depth, S is the gradient, g is the acceleration due to gravity, and $\bar{v}$ is the mean flow speed. In the derivation of this relationship (e.g. White, 1999), the depth term is intended to reflect only the dynamic mutual adjustment that exists between slope gradient, flow speed and flow depth in steady flows traversing an inclined surface with properties that give rise to the frictional retardation expressed in the f term. A streamwise change in depth arising from other factors (such as altered volumetric displacement along the flow path) would make Equation 1 inapplicable.

The volume that obstacles occupy below the water surface is involved in depth-related changes in f. If this volume becomes large, it may significantly displace the flow depth upwards. If the obstacles remain only partially immersed, then this displacement would reduce f in laminar flows but increase it in turbulent flows. If volumetric displacement caused the obstacles to be overtopped, then in both flow regimes, f would decline. Measured values of f for flows past fields of obstacles therefore reflect a variable influence arising from the volumetric displacement of flow depth. It is argued below that to answer some research questions (illustrated next), a volumetric displacement component should be deducted from measured flow depths to allow the determination of an adjusted flow depth and an adjusted Darcy–Weisbach f, here denoted d^* and f^* respectively. Ideally, analytical treatments of frictional retardation should be capable of treating the volumetric displacement of depth separately from the effects on depth of dynamic, frictional flow retardation. This becomes crucial in understanding the role of surface cover fraction (e.g. of crop residue, litter, or stones) in influencing f. In experiments where varying cover fractions of such materials are related to the value of f (e.g. Gilley *et al.*, 1990, 1992), there is a confounding of two mechanisms. The increasing abundance of obstacles as cover fraction increases is one mechanism, perhaps leading to greater obstruction of flow, and/or greater tortuosity of flow paths, and hence to a real increase in frictional retardation. However, these effects are intertwined with the increasing volumetric displacement that would arise on a particular test surface carrying larger numbers of obstacles. Any depth increase arising from this effect ought not to be incorporated into estimates of f as derived from Equation 1. Indeed, the addition of sufficient additional obstacles to a flow that was already nearly submerging an existing obstacle field could result in the depth being displaced upward so that all of the obstacles were overtopped by the flow. In this case, the addition of obstacles could *reduce* the measured friction coefficients, which is not what physical intuition suggests ought to be the outcome of increasing the abundance of obstacles. The true effect of the varying cover fraction could be revealed therefore only by eliminating the confounding influence of volumetric displacement. A second point that has potential significance for the parameterization of soil surface properties in overland flow modelling is that the effect on flow depth exerted by the dynamic frictional drag and volumetric displacement effects may relate to different aspects of obstacles within the flow (such as upslope-projected wetted area or submerged volume).

The need for a correction for submerged obstacle volume was noted previously by Turner *et al.* (1978), who studied flow through simulated grass. They proposed the use of an 'equivalent depth' of flow, which was the depth found when the volume of submerged stems was deducted from the total volume of water plus stems. Turner *et al.* (1978) found that for their experimental conditions, this correction amounted to only about 10 per cent of flow depth, and did not proceed with the development of revised friction factors. No subsequent work is known to the writer.

Exploring the need for a volumetric displacement correction in measures of flow depth

Changes in the volumetric displacement of flow depth warrant separate recognition, especially because they may contribute variability to the form of the f–Re relationship as discharge through a field of obstacles is varied, or as the density of obstacles is varied for a given flow rate. Consider for example the experiments of Rauws (1988) in which regular hemispherical obstacles 1·6 cm in diameter were glued to the bed of a flume and exposed to flows of 9·4–472 cm^3 s^{-1}. These obstacles would have their greatest proportional volumetric displacement effect on water levels in shallow flows, when the diameter of the wetted part of the hemisphere was greatest. The relative importance of the volumetric displacement effect would then decline to an approximately constant value once the tops of the hemispheres were covered by the flow. Thus, for flow depths varying through this interval, the displacement effect would have diminishing importance and the true frictional drag on the bed and the obstacles an increasing role in determining flow depth. Without quantifying these effects, it is not possible to apportion the depth changes produced when imposed flow rate into the flume was varied between displacement and increasing obstacle area facing the flow. Nevertheless, Rauws (1988), following the experimental demonstration by Einstein and Banks (1950) that different sources of roughness can be additive, adopted the relationship

$$d = d' + d'' \tag{2}$$

where d is flow depth, d' is the flow depth related to grain resistance, and d'' is the flow depth due to form resistance. This relationship neglects the effect of volumetric displacement, so that a more complete model is

$$d = d' + d'' + d''' \tag{3}$$

where d''' is the depth related to volumetric displacement. Under this scheme, the part of the depth relating to grain and form resistance would differ from the values reported by Rauws (1988). More importantly, the difference could not remain constant but would co-vary with depth (or equivalently, Q or Re of the imposed flow).

Rauws (1988) adopted a related expression for the components of f:

$$f = f' + f'' \tag{4}$$

which is again modified here to take the form

$$f = f' + f'' + f''' \tag{5}$$

in which f''' is the component of the friction coefficient related to the effects of volumetric displacement. This term may carry a positive or negative sign, depending upon whether the volumetric displacement increases f as a result, for instance, of a growing upslope-projected obstacle area, or diminishes f because obstacles are overtopped in consequence of the depth displacement, or because they increase the value of Re of laminar flows bypassing obstacles with an associated decline in the drag coefficient.

In summary, significant volumetric displacement of water depth is an inherent aspect of shallow flow through obstacle fields, and in one sense can validly be regarded as contributing to the value of f exhibited by the flow and estimated in accordance with Equation 1. However, unless the obstacles are all regular prisms, for which the incremental increase in submerged volume is a linear function of flow depth, the volumetric displacement effect can produce irregular shifts in flow depth that have the potential to confound efforts to resolve the effects of other factors such as the spacing, size, shape, or other aspects of drag related to the presence of obstacles. A procedure to eliminate the volumetric displacement effect therefore would assist in the understanding of the hydraulics of shallow overland flows.

Outline of paper

In the remainder of this paper, new experimental data on shallow laminar flows passing through fields of protruding obstacles are used to attempt a separation of true obstacle drag effects on flow depth from those of volumetric displacement. Attention is focused on flow depth because it is more readily measurable than hydraulic radius, and so has become the standard used in field and laboratory studies of shallow overland flow. The primary goal of the present work is to derive an approach to the derivation of f from Equation 1 when volumetric displacement is present. Only laminar flow is analysed in the first series of experiments reported here; turbulent flows exhibit different behaviour and will require separate investigation. Laminar overland flows are important and spatially extensive in the interrill zone (Woolhiser *et al.*, 1970). Although the new experimental results apply only to laminar flows, the principle outlined here, and the need for the correction of measured flow depths to account for volumetric displacement, applies equally to all shallow overland flows affected by obstacles, regardless of the flow regime.

DEVELOPING A CORRECTED MEASURE OF FLOW DEPTH, d

Flow depth (d) in an obstacle field is the product of both a dynamic, frictional component and volumetric displacement, and both factors contribute to increased flow depth in comparison with the same flow with no obstacles. In the shallow flow experiments described later, the additional depth attributable to volumetric displacement (d''') is known to be 20 per cent of the depth measured in an obstacle-free flow of the same discharge on a bare flume bed roughened with sand grains (d_{sand}), because in all experiments this is the proportion of the flow volume occupied by obstacles of fixed dimensions (square ceramic tiles are used as obstacles). The remaining component of the greater depth in obstacle flow (d'') is the fraction attributable to actual form drag. That is,

$$d'' = (d - d_{\text{sand}}) - d''' \tag{6}$$

For a given discharge, the corrected obstacle flow depth d^* (i.e., with volumetric displacement effects eliminated) is then found as the sum of the depth measured in obstacle-free flow (d_{sand}) at the same discharge, and the additional depth attributable to obstacle drag, d''. The volumetric displacement term d''' is thus eliminated. Hence,

$$d^* = d_{\text{sand}} + d'' \tag{7}$$

In what follows, terms marked with an asterisk (*) refer to friction coefficients or flow depths from which the volumetric displacement effect has been removed. Thus, in the more general symbols used earlier, Equation 7 can equivalently be written $d^* = d'^* + d''^*$. To improve clarity, the subscripts 'sand' and 'tile' are added to key terms in equations that follow, to indicate whether that term is derived from obstacle-free flow on the sand board, or flow through the obstacles.

Having found d^*, the grain and obstacle roughness can be found. The grain roughness arising on the bed between the obstacles (f'^*_{sand}) in a flow of depth d^* is found using an $f - d$ power-function relationship established on the sand board with no obstacles present, and solved for the depth d^*. The use of a regression model linking f and d is needed because, in general, d^* does not correspond to any of the actual flow depths arising from the imposed discharges. The fitted relationships were of the form

$$f = ad^b \tag{8}$$

where a and b are fitted by least-squares. All fitted relationships were statistically significant at levels of $\alpha = 0{\cdot}05$ or better.

The value of f^*_{tile} for obstacle flow (the friction coefficient exhibited in the obstacle field when the flow depth is d^*) is obtained similarly from an $f - d$ power-function relationship developed in the obstacle fields, and again solved at the depth d^*.

Finally, the obstacle drag (f''^*_{tile}) at the depth d^* is found by subtraction. That is,

$$f''^*_{\text{tile}} = f^*_{\text{tile}} - f'^*_{\text{sand}} \tag{9}$$

Finally, at each discharge f''' is found by taking the difference between the unadjusted value of f_{tile} as determined for the obstacle field using Equation 1, and the value corrected for volumetric displacement, f^*_{tile}:

$$f''' = f_{\text{tile}} - f^*_{\text{tile}} \tag{10}$$

Hence we can write

$$f_{\text{tile}} = f'^{*}_{\text{sand}} + (f^{*}_{\text{tile}} - f'^{*}_{\text{sand}}) + f''' \tag{11}$$

so that Equation 5 reduces to

$$f_{\text{tile}} = f^{*}_{\text{tile}} + f''' \tag{12}$$

This shows clearly that the f''' term expresses the difference in friction coefficient (f_{tile}) derived from unadjusted flow data and the value corrected for volumetric displacement (f^{*}_{tile}).

EXPERIMENTAL METHODS

Experiments were conducted on a board 0·5 m wide and 1·2 m long, coated with medium sand that was fixed by application to a coat of uncured varnish, and set at a fixed gradient of 1·2 . Imposed discharges were fed to the top of the board through a perforated pipe, and were varied in five increments for the sand board with no emplaced obstacles, and four increments for each of two obstacle fields. The range of flow rates used was 10 cm^3 s^{-1} to 50 cm^3 s^{-1}, and variations in pump efficiency meant that identical flow rates could not be achieved in each experiment. Actual flow rates were determined by repeated volumetric gauging of the discharge passing into a collecting gutter below the lower end of the board. The behaviour of laminar flows on the sand board was studied, and then a 20 per cent cover of obstacles (glazed ceramic tiles) was set randomly in place and the same range of flows remeasured. In separate experiments, two sizes of obstacle were used (small, 25 × 25 mm; large 46 × 46 mm). The tiles were 6 mm in thickness and were never overtopped by the flow. A computer-controlled X-Y gantry carrying a precision stepping needle-point gauge recorded water surface elevations, via closure of an electrical circuit, at a grid of 64 test points in a central 0·25 m^2 area of the sand board. The surface elevation was resolved with a resolution of 25 m. The same device was used to measure the elevations of the dry sand board, again with a resolution of 25 m, and flow depths were found by subtraction. Water temperature was recorded using a digital platinum resistance thermometer, to allow viscosity to be calculated. Flow width was taken, following the procedure of Abrahams and Parsons (1990), as 0·8 of the total board width, because 0·2 of the width was taken up by non-submerged obstacles and carried no flow except for a small amount, estimated at 0·4 per cent, that seeped slowly beneath the obstacles as they rested on the roughened sand surface. This small flow is neglected. The experimental apparatus is described more fully in Dunkerley (2001) and Dunkerley *et al.* (2001).

The Reynolds number *Re* was computed from the relationship

$$Re = \frac{4dv}{\nu} \tag{13}$$

where d is mean flow depth (cm), v is mean flow velocity (cm s^{-1}), and ν is the kinematic viscosity of the water (cm^2 s^{-1}). The Froude number was computed from

$$F = \frac{v}{\sqrt{gd}} \tag{14}$$

Surface velocities were determined by dye-arrival timing, but these cannot be used to reliably infer flow-averaged velocities (Dunkerley, 2001). Therefore, the velocity required for the use of Equation 1 was estimated from $v = \frac{Q}{wd}$.

RESULTS

All flows were laminar and subcritical, with $104 < Re < 490$ and $0{\cdot}33 < F < 0{\cdot}75$ (Table I). Water temperatures lay in the range 18 –20 C, and viscosity changed little between experiments. Flow depths in the obstacle

Table I. Summary of test conditions on the sand board, and in the small and large obstacle fields

Test condition	Range of imposed discharges (cm^3 s^{-1})	Range of flow depths (cm)	Range of flow speeds (cm s^{-1})	Range of Froude Numbers (F)	Range of Reynolds numbers (Re)	Obstacle cover fraction (per cent)
Bare sand board	12·6–50·1	0·085–0·122	2·98–8·23	0·33–0·75	104–410	0
Large obstacle field	14·9–40·0	0·103–0·138	3·62–7·25	0·36–0·62	141–380	20
Small obstacle field	13·9–48·3	0·10–0·164	3·47–7·34	0·35–0·58	141–490	20

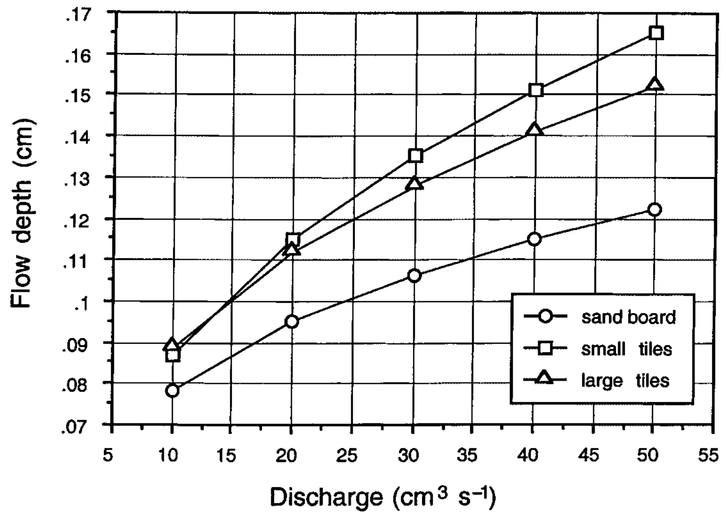

Figure 1. Flow depth on the bare sand board, and the sand board with small and large obstacles, as a function of imposed discharge. The greater flow depths arising in the obstacle fields are evident, as is the greater increment of flow depth over that in obstacle-free flow caused by small obstacles. Both small and large obstacle fields had the same 20 per cent cover fraction

fields generally were more than 20 per cent larger than those on the bare sand board at the same flow rate (Figure 1). However, in the smallest flows tested (12–14 cm^3 s^{-1}) depths were greater on the bare sand board than in obstacle flows. Separate experiments showed this to relate to the draw-up of water into curving menisci around the obstacle margins, which results in a decline in depth in the spaces between the obstacles. This small effect is most evident in very shallow flows, and is the subject of a separate paper (Dunkerley, In press). Because of the perturbing effects of meniscus formation, results from tests at the lowest flow rate are not included in some of the analyses of the experimental data that follow, as these results diverge from the consistent ones found at all higher flow rates. Despite the influence of surface tension in some of the very shallow flows, Moody plots were consistently negatively sloping, as expected in the laminar flow regime (Figure 2).

A single example will illustrate the procedure used in the adjustment of the friction coefficients to make allowance for volumetric displacement of flow depth. Consider an imposed flow of 30 cm^3 s^{-1} passing through the field of large obstacles (Table II; Table III provides the corresponding results for the small obstacle tests). The unadjusted f derived from Equation 1 was 0·628, little higher than the value on the bare sand board for the same flow, which was 0·550. However, the value of f^* was 1·060 (as exhibited in obstacle flows at the reduced depth d^*), whereas f'^* was 0·546; this results in a value of f''^* of 0·514. Thus, Equation 5 can now be written

$$0{\cdot}628 = 0{\cdot}546 + 0{\cdot}514 + f''' \quad (15)$$

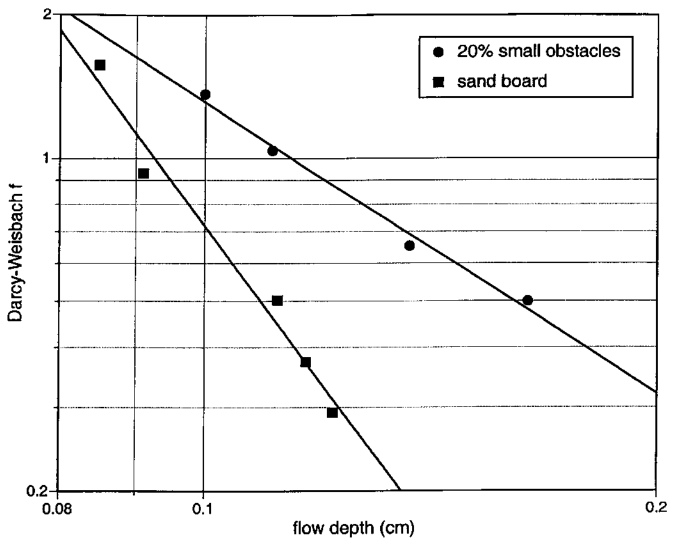

Figure 2. Relationships between flow depth and the Darcy–Weisbach friction factor f for the bare sand board and for the field of small obstacles. Details of the power function regression models shown as straight lines are provided in the text

Table II. Discharge and Darcy–Weisbach f details for flow through the large obstacle field. The symbols used are explained in the text

Discharge (cm^3 s^{-1})	d (cm)	d^* (cm)	f' (sand board)	f (obstacle field)	f^*	f'^*	f''^*	f'''
10·0	0·089	0·073	1·968	1·787	3·129	2·681	0·448	−1·342
20·0	0·112	0·093	0·881	0·922	1·577	0·979	0·598	−0·655
30·0	0·128	0·107	0·550	0·628	1·060	0·546	0·514	−0·432
40·0	0·141	0·118	0·394	0·476	0·795	0·357	0·437	−0·319
50·0	0·152	0·127	0·304	0·383	0·635	0·257	0·378	−0·252

Table III. Discharge and Darcy–Weisbach f details for flow through the small obstacle field. The symbols used are explained in the text

Discharge (cm^3 s^{-1})	d (cm)	d^* (cm)	f' (sand board)	f (obstacle field)	f^*	f'^*	f''^*	f'''
10·0	0·087	0·071	1·968	1·733	2·593	3·014	−0·422	−0·860
20·0	0·115	0·096	0·881	0·987	1·423	0·856	0·567	−0·436
30·0	0·135	0·114	0·550	0·714	1·010	0·417	0·593	−0·296
40	0·151	0·128	0·394	0·570	0·796	0·253	0·543	−0·226
50·0	0·165	0·140	0·304	0·477	0·659	0·171	0·489	−0·182

so that f''' must be −0·432. The negative result indicates that frictional retardation has been *lowered* by the upward displacement of depth, as would be expected in laminar flows. In this case, f^* is seen to exceed f by 69 per cent.

ANALYSING THE COMPONENTS OF f

For the bare sand board, the fitted relationship between f and flow depth d (cm) was

$$f = 4{\cdot}24 \times 10^{-5} d^{-4{\cdot}229} \quad (16)$$

for which $r^2 = 0{\cdot}98$. The relationship is statistically significant ($p = 0{\cdot}0016$).

The relationship between imposed discharge Q (cm^3 s^{-1}) and flow depth (cm) on the sand board, and used to determine the depth that would occur there for the various discharges imposed on the obstacle fields, was

$$d = 0{\cdot}0419 Q^{0{\cdot}2743} \quad (17)$$

This is a statistically significant relationship ($p < 0{\cdot}001$) for which $r^2 = 0{\cdot}99$.

Separate comments are provided below for large obstacle and small obstacle tests, both of which were based on obstacle fields set out on the sand board just described.

Large obstacles

Results for the large ceramic tile experiments show that for a surface cover fraction of 20 per cent, the correction for volumetric displacement resulted in values of f^* that were 65–75 per cent greater than the unadjusted value of f derived from Equation 1. For flows up to about 30 cm^3 s^{-1}, bed grain roughness (f'^*) exceeded the obstacle roughness (f''^*) and was also greater than the unadjusted roughness f. For a flow of 20 cm^3 s^{-1}, for instance, f on the bare sand board was 0·881, whereas the same flow rate through a 20 per cent obstacle cover yielded f of only 0·922. The small increase in roughness for the obstacle flow is accounted for by the value of f''' of $-0{\cdot}655$. The value of f''' was negative for all flow rates. For the smallest flow rate tested, the upward displacement of flow depth thus reduced the overall roughness of flow through the obstacles to a lower value than was recorded in the absence of the obstacles. The fitted form of Equation 8, relating f to depth d (cm) was

$$f = 0{\cdot}0017 d^{-2{\cdot}876} \quad (18)$$

($r^2 = 0{\cdot}89$, statistically significant at $p < 0{\cdot}05$).

Small obstacles

For a 20 per cent cover of small obstacles, the correction for volumetric displacement resulted in values of f^* that were 38–50 per cent greater than the unadjusted value of f derived from Equation 1. As noted for large tiles, bed roughness (f'^*) exceeded obstacle roughness (f''^*) for flows up to about 20 cm^3 s^{-1}. For small obstacles, f''' was negative for all flow rates. Higher flow rates behaved as for the large obstacles, obstacle roughness f''^* exceeding grain roughness f'^*.

The fitted form of Equation 8, relating f to depth d (cm) was

$$f = 0{\cdot}0126 d^{-2{\cdot}016} \quad (19)$$

($r^2 = 0{\cdot}99$, statistically significant at $p < 0{\cdot}005$).

Differences between large and small obstacle tests

Despite the finding of consistently negative values of f''' in both large and small obstacle tests, there were some differences in the absolute values of the roughness terms. For the same discharge, flow depths were larger in the small obstacle field than in the large obstacle field. Similarly, for an equal discharge or flow depth, roughness f was larger in the small obstacle field than in the large obstacle field. An interpretation of these findings is offered shortly.

DISCUSSION

The present findings show that making a correction for volumetric displacement of flow depth by stone-like obstacles standing above the elevation of an essentially planar bed can result in significantly changed estimates of the Darcy-Weisbach friction coefficient. In the absence of a volumetric displacement correction, obstacle roughness is underestimated at all flow rates. For the obstacle cover tested (20 per cent), the unadjusted value of f was only 60–70 per cent of the corrected value f^*. This resulted in the unadjusted value of f for flow through the obstacle field being only slightly higher than for the same flow rate on the bare sand board for the higher flows, and actually lower for flow past obstacles than for the bare sand board, at low flow rates. Thus, at low flow rates, the anticipated greater roughness arising from the obstacle field is concealed unless a volumetric displacement correction is applied.

The obstacle densities tested here fall within the range of surface stone covers found in many dryland areas. For example, Roels (1984) studied flow on field plots carrying up to 25 per cent cover of limestone fragments. However, stone covers of >80 per cent are not uncommon on desert surfaces. Such very high surface covers have been applied in a number of investigations of flow friction factors. Gilley *et al.* (1992) performed flume tests in which the bed carried up to 83 per cent cover of gravel, and Abrahams *et al.* (1986) studied runoff plots in Arizona that had 60–80 per cent cover of stones >2 mm in diameter. Other studies have dealt with flow past obstacle fields, including the work of Rauws (1988) referred to earlier. In all of these studies, values for f were derived from flows through and over the obstacles, but no correction was applied for the volume that they occupied below the water surface. Each experiment therefore contains a unique degree of perturbation of the results by volumetric displacement, and this prohibits direct comparison of the various results dealing with frictional retardation in the various flows studied.

Changes in the displacement term f''' with flow depth

Earlier, the effect of changing flow rate on the volumetric displacement of depth by hemispheric obstacles (Rauws, 1988) was noted. Similarly, Gilley *et al.* (1992) studied flow across varying cover fractions of stones and cobbles set on the bed of a flume in which the number of obstacles on the bed of the flume was varied to give cover fractions ranging from 5 to 95 per cent. All bed conditions were exposed to the same range of imposed discharges. In these experiments, all three of the friction factor components in Equation 5 must have changed among the experimental runs. Without a separate analysis, it is not immediately clear which tests involved large volumetric displacements and which lesser ones. Consequently, the true change in frictional retardation arising from the changing obstacle cover remains unresolved, because this effect is confounded with the changing volumetric displacement. That is, some component of the change in f' and f'' would in reality have arisen from variation in f'''.

An analysis of volumetric displacement by spheres of uniform and mixed sizes

In the ceramic tile tests made in the present work, the volume below the water surfaces changes by a constant amount with each unit change in flow depth. In contrast, obstacles of other shapes (such as irregularly rounded surface stones, or pieces of stem-like crop residue) would behave in more variable ways. Obstacles of mixed sizes also would lead to different results.

Take for example the ideal case of spherical 'stones' of uniform diameter resting on a granular surface. At low flow rates, only small fractions of each spherical stone might be submerged; the volume occupied below the water surface would then rise at an increasing rate with flow depth until the spheres were covered more deeply, after which the rate of increase in submerged volume would decline, reaching its maximum when the spheres were just covered by the flow. Geometric analysis of the aggregate volume below the water surface shows readily that as depth increases, submerged volumes follow a sigmoidal curve. Although this curve would not apply to irregular natural stones, it provides an indication of the effect on flow depths. The volumetric displacement of the water surface in such a situation is portrayed in Figure 3 for a 20 per cent cover fraction of uniform spherical obstacles.

This graph reveals that in the case of uniform spheres, the partial volume taken up below the water surface peaks when the spheres are about three-quarters submerged (actually to a depth of 0·74 diameters) and then begins to decline. Natural stone veneers are not composed of stones of uniform size, but rather include a

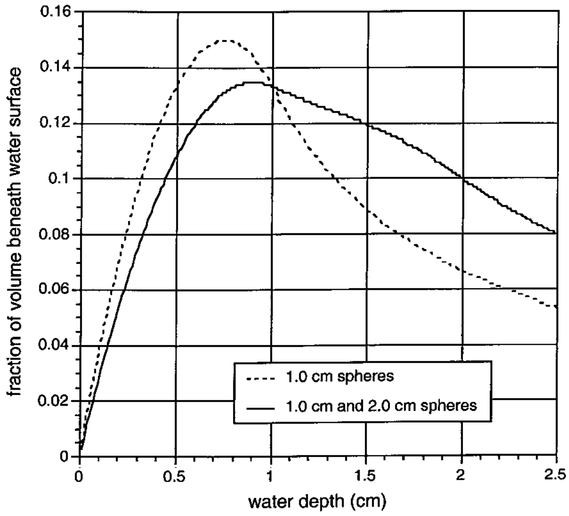

Figure 3. Dependence of the volume fraction beneath the water surface occupied by obstacles as a function of the water depth. Two cases are shown: a 20 per cent cover of uniform, 1 cm diameter spheres (dashed line), and a 20 per cent cover composed of 10 per cent of 1 cm diameter spheres and 10 per cent of 2 cm diameter spheres (solid line)

range of sizes (Dunkerley, 1995). An additional simple geometric analysis demonstrates that mixtures of stone sizes can have the effect of changing the flow depth at which maximum volume fraction occupied below the water surface is reached.

Figure 3 also shows the simple case of a 20 per cent surface cover made up of 10 per cent each of 1 cm and 2 cm diameter stones (a cover requiring 1274 of the smaller and 318 of the larger stones per m^2). Here it can be seen that the fraction of the volume below the water surface occupied by the stones peaks at around 0·9 small-stone diameters and then begins to decline. Thus, by providing increasingly large submerged volume as the flow approaches the tops of the small stones, the larger ones distort the volume–water-level relationship. Additionally, the maximum volume fraction occupied by the spheres beneath the water surface is lowered by about 0·015 in the presence of the two sizes of sphere. Thus, the largest error in the unadjusted value of the Darcy–Weisbach f can arise at about 0·74 obstacle diameters for uniform spheres, but at nearly 1 diameter when the numerically dominant smaller spheres are mixed with fewer larger ones, in a mixed-size obstacle field. This co-variation of volumetric displacement with flow depth is more like the form that would be expected in overland flow traversing a soil surface carrying a natural, log-normally distributed population of surface stones.

In the case of field studies where the obstacles (surface stones, litter, stems) are of irregular size and shape, the correction for volumetric displacement would require minor additional experimentation. The simplest approach probably would be to impound a small, representative area of the obstacle field. This could be done with an infiltration cylinder, for example, embedded to a shallow depth. The soil surface would be made impermeable with a spray lacquer. Adding to this impoundment known small volumes of water and recording the resulting depth in the test cylinder would allow the volumetric displacement profile to be recorded. That is, for each ponding depth, the corresponding obstacle volume below the water surface could be derived. These results would then allow the volumetric displacement to be corrected in data on experimental flows of any particular depth through the obstacle field.

CONCLUSIONS

Obstacles partially or wholly submerged in shallow flows occupy part of the volume under the water surface. In the case of protruding obstacles in laminar flows, the resulting upward displacement of the water surface and the concentration of the flow into the restricted area between the obstacles tend to decrease the value of f calculated from Equation 1. Because the extent of volumetric displacement of water depth depends upon the shapes, sizes and cover fraction of the obstacles, it provides a confounding influence in studies aimed at resolving differences in f related to other aspects of the flow environment. Studies of flows passing through natural obstacle fields such as vegetation, litter, or natural desert stone veneers, cannot be compared one with another owing to the differing extent of volumetric displacement in each study.

The presence of a significant volumetric displacement effect on flow depths in shallow flows also causes complications in studies aimed at showing how friction coefficients vary with flow rate. This is because obstacles of different shape occupy changing proportions of the volume below the water surface as depth varies. For spherical obstacles (approximated perhaps by many surface stones) the proportion of volume occupied is small at shallow depths, rises to a maximum at about 0·74 obstacle diameters, and then declines. However, when the sorting of obstacle sizes is varied, the depth where maximum displacement occurs can vary from shallower to deeper flow depths. Thus, the volume displacement effect can act as a confounding influence co-varying with flow rate.

The results presented here also demonstrate that for laminar flows, the volumetric displacement arising from the submerged parts of obstacles can lead to substantial underestimation of the value of f.

Although the experimental data presented here come from laminar flows, related kinds of perturbation of friction factors can be expected in turbulent flows. The nature of the changes in f would, however, be somewhat different because of the general increase in drag coefficients with depth (or Re) that arises in turbulent flows through obstacle fields, which contrasts with the falling drag coefficients at higher values of Re that arise in laminar flows. Certainly, an examination of the kinds of volumetric displacements described here for the case of turbulent flows seems necessary. The adoption of a correction for volumetric displacement is recommended for future investigations of friction factors in shallow flows containing obstacles, as this would serve both to provide additional evidence on the magnitude and importance of the effect under varying surface roughness and flow conditions, and at the same time provide a means of standardizing experimental results in order to make them more directly comparable.

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REFERENCES

Abrahams AD, Parsons AJ. 1990. Determining the mean depth of flow in field studies of flow hydraulics. *Water Resources Research* **26**: 501–503.

Abrahams AD, Parsons AJ. 1994. Hydraulics of interrill overland flow on stone-covered desert surfaces. *Catena* **23**: 111–140.

Abrahams AD, Parsons AJ, Luk S-H. 1986. Resistance to overland flow on desert hillslopes. *Journal of Hydrology* **88**: 343–363.

Dunkerley DL. 1995. Surface stone cover on desert hillslopes: parameterizing characteristics relevant to infiltration and surface runoff. *Earth Surface Processes and Landforms* **20**: 207–218.

Dunkerley DL. 2001. Estimating the mean speed of laminar overland flow using dye injection – uncertainty on rough surfaces. *Earth Surface Processes and Landforms* **26**: 363–374.

Dunkerley DL. In press. Surface tension and friction coefficients in shallow, laminar overland flows through organic litter. *Earth Surface Processes and Landforms*.

Dunkerley DL, Domelow P, Tooth D. 2001. Frictional retardation of laminar flow by plant litter and surface stones on dryland surfaces: a laboratory study. *Water Resources Research* **37**: 1417–1424.

Einstein HA, Banks RB. 1950. Fluid resistance of composite roughness. *Transactions of the American Geophysical Union* **31**: 603–610.

Gilley JE, Kottiwtz ER, Wieman GA. 1990. Roughness coefficients for selected residue materials. *Journal of Irrigation and Drainage Engineering* **117**: 503–514.

Gilley JE, Kottwitz ER, Wieman GA. 1992. Darcy–Weisbach roughness coefficients for gravel and cobble surfaces. *Journal of Irrigation and Drainage Engineering* **118**: 104–112.

Rauws G. 1988. Laboratory experiments on resistance to overland flow due to composite roughness. *Journal of Hydrology* **103**: 37–52.

Roels JM. 1984. Flow resistance in concentrated overland flow on rough slope surfaces. *Earth Surface Processes and Landforms* **9**: 541–551.

Turner AK, Langford KJ, Win M, Clift TR. 1978. Discharge–depth equation for shallow flow. *Journal of the Irrigation and Drainage Division, Proceedings of the American Society of Civil Engineers* **104**(IR1): 95–110.

White FM. 1999. *Fluid Mechanics*. McGraw-Hill: Singapore; 4th edn. 826 pp.

Woolhiser DA, Hanson CL, Kuhlman AR. 1970. Overland flow on rangeland watersheds. *Journal of Hydrology, New Zealand* **9**: 336–356.

