

Use the given dictionary to translate the following argument:

A person cannot be guilty of murder unless they are capable of understanding that their action caused the death of another person. But no one who is capable of understanding that their action caused the death of another person is insane. Consequently, no one who is guilty of murder is insane.

Px - x is a person

Gx - x is guilty of murder

Ux - x is capable of understanding that their action caused the death of another person.

Ix - x is insane

<p>A.*</p> <p>1. $(\forall x)((Px \ \& \ Gx) \supset Ux)$</p> <p>2. $(\forall x)((Px \ \& \ Ux) \supset \sim Ix)$</p> <p>Therefore:</p> <p>C. $(\forall x)((Px \ \& \ Gx) \supset \sim Ix)$</p>	<p>B.</p> <p>1. $(\forall x)((Px \ \& \ Gx) \supset Ux)$</p> <p>2. $\sim(\exists x)((Px \ \& \ Ux) \supset \sim Ix)$</p> <p>Therefore:</p> <p>C. $\sim(\exists x)((Px \ \& \ Gx) \supset \sim Ix)$</p>
<p>C.</p> <p>1. $(\forall x)((Px \ \& \ Gx) \supset Ux)$</p> <p>2. $\sim(\forall x)((Px \ \& \ Ux) \supset Ix)$</p> <p>Therefore:</p> <p>C. $\sim(\forall x)((Px \ \& \ Gx) \supset Ix)$</p>	<p>D.</p> <p>1. $(\forall x)((Px \ \& \ Gx) \supset Ux)$</p> <p>2. $\sim(\exists x)((Px \ \& \ Ux) \ \& \ \sim Ix)$</p> <p>Therefore:</p> <p>C. $\sim(\exists x)((Px \ \& \ Gx) \ \& \ \sim Ix)$</p>

$(\forall x)((Px \ \& \ Gx) \supset Ux), (\forall x)((Px \ \& \ Ux) \supset \sim Ix) \vdash (\forall x)((Px \ \& \ Gx) \supset \sim Ix)$

We will test the argument for validity using trees. How does the tree begin?

<p>A.</p> <p>$(\forall x)((Px \ \& \ Gx) \supset Ux)$ $(\forall x)((Px \ \& \ Ux) \supset \sim Ix)$ $(\forall x)((Px \ \& \ Gx) \supset \sim Ix)$</p>	<p>B.*</p> <p>$(\forall x)((Px \ \& \ Gx) \supset Ux)$ $(\forall x)((Px \ \& \ Ux) \supset \sim Ix)$ $\sim(\forall x)((Px \ \& \ Gx) \supset \sim Ix)$</p>
<p>C.</p> <p>$\sim(\forall x)((Px \ \& \ Gx) \supset Ux)$ $\sim(\forall x)((Px \ \& \ Ux) \supset \sim Ix)$ $(\forall x)((Px \ \& \ Gx) \supset \sim Ix)$</p>	<p>D.</p> <p>$\sim(\forall x)((Px \ \& \ Gx) \supset Ux)$ $\sim(\forall x)((Px \ \& \ Ux) \supset \sim Ix)$ $\sim(\forall x)((Px \ \& \ Gx) \supset \sim Ix)$</p>

1. $(\forall x)((Px \ \& \ Gx) \supset Ux)$
2. $(\forall x)((Px \ \& \ Ux) \supset \sim Ix)$
3. $\sim(\forall x)((Px \ \& \ Gx) \supset \sim Ix)$

What rule should we apply next?

- A. The rule for \forall to line 1.
- B. The rule for \forall to line 2.
- C. Either A or B, it makes no difference.
- D.* The rule for $\sim \forall$ to line 3.

What does the next stage of the tree look like?

<p>A.</p> $ \begin{array}{l} (\forall x)((Px \ \& \ Gx) \supset Ux) \\ (\forall x)((Px \ \& \ Ux) \supset \sim lx) \\ \sim(\forall x)((Px \ \& \ Gx) \supset \sim lx) \ \checkmark a \\ \quad \quad \quad \\ ((Pa \ \& \ Ga) \supset \sim la) \end{array} $	<p>B.*</p> $ \begin{array}{l} (\forall x)((Px \ \& \ Gx) \supset Ux) \\ (\forall x)((Px \ \& \ Ux) \supset \sim lx) \\ \sim(\forall x)((Px \ \& \ Gx) \supset \sim lx) \ \checkmark a \\ \quad \quad \quad \\ \sim((Pa \ \& \ Ga) \supset \sim la) \end{array} $
<p>C.</p> $ \begin{array}{l} (\forall x)((Px \ \& \ Gx) \supset Ux) \\ (\forall x)((Px \ \& \ Ux) \supset \sim lx) \\ \sim(\forall x)((Px \ \& \ Gx) \supset \sim lx) \ \checkmark a \\ \quad \quad \quad \\ \sim((Pa \ \& \ Ga) \supset la) \end{array} $	<p>D.</p> $ \begin{array}{l} (\forall x)((Px \ \& \ Gx) \supset Ux) \\ (\forall x)((Px \ \& \ Ux) \supset \sim lx) \\ \sim(\forall x)((Px \ \& \ Gx) \supset \sim lx) \ \checkmark a \\ \quad \quad \quad \\ Pa \ \& \ Ga \\ \quad \quad \quad \sim la \end{array} $

1. $(\forall x)((Px \ \& \ Gx) \supset Ux)$
2. $(\forall x)((Px \ \& \ Ux) \supset \sim lx)$
3. $\sim(\forall x)((Px \ \& \ Gx) \supset \sim lx) \ \checkmark \ a$
- |
4. $\sim((Pa \ \& \ Ga) \supset \sim la)$

What rule should we apply next?

- A. The rule for \forall to line 2.
- B.* The rule for $\sim\supset$ to line 4.
- C. The rule for \forall to line 1.
- D. The rule for $\sim\forall$ to line 3.

What does the next stage of the tree look like?

<p>A.*</p> $ \begin{array}{l} (\forall x)((Px \ \& \ Gx) \supset Ux) \\ (\forall x)((Px \ \& \ Ux) \supset \sim Ix) \\ \sim(\forall x)((Px \ \& \ Gx) \supset \sim Ix) \ \checkmark \ a \\ \quad \\ \quad \sim((Pa \ \& \ Ga) \supset \sim Ia) \ \checkmark \\ \quad \quad \\ \quad \quad Pa \ \& \ Ga \\ \quad \quad \quad \\ \quad \quad \quad \sim \sim Ia \ \checkmark \\ \quad \quad \quad \quad \\ \quad \quad \quad \quad Ia \end{array} $	<p>B.</p> $ \begin{array}{l} (\forall x)((Px \ \& \ Gx) \supset Ux) \\ (\forall x)((Px \ \& \ Ux) \supset \sim Ix) \\ \sim(\forall x)((Px \ \& \ Gx) \supset \sim Ix) \ \checkmark \ a \\ \quad \\ \quad \sim((Pa \ \& \ Ga) \supset \sim Ia) \ \checkmark \\ \quad \quad / \quad \backslash \\ \quad \quad \sim(Pa \ \& \ Ga) \quad \sim \sim Ia \ \checkmark \\ \quad \quad \quad \quad \\ \quad \quad \quad \quad Ia \end{array} $
<p>C.</p> $ \begin{array}{l} (\forall x)((Px \ \& \ Gx) \supset Ux) \\ (\forall x)((Px \ \& \ Ux) \supset \sim Ix) \\ \sim(\forall x)((Px \ \& \ Gx) \supset \sim Ix) \ \checkmark \ a \\ \quad \\ \quad \sim((Pa \ \& \ Ga) \supset \sim Ia) \ \checkmark \\ \quad \quad / \quad \backslash \\ \quad \quad \sim \sim (Pa \ \& \ Ga) \quad \sim Ia \end{array} $	<p>D.</p> $ \begin{array}{l} (\forall x)((Px \ \& \ Gx) \supset Ux) \\ (\forall x)((Px \ \& \ Ux) \supset \sim Ix) \\ \sim(\forall x)((Px \ \& \ Gx) \supset \sim Ix) \ \checkmark \ a \\ \quad \\ \quad \sim((Pa \ \& \ Ga) \supset \sim Ia) \ \checkmark \\ \quad \quad \\ \quad \quad Pa \ \& \ Ga \\ \quad \quad \quad \\ \quad \quad \quad \sim Ia \end{array} $

How should the tree continue?

<p>A.</p> $(\forall x)((Px \ \& \ Gx) \supset Ux) \ \backslash b$ $(\forall x)((Px \ \& \ Ux) \supset \sim Ix)$ $\sim(\forall x)((Px \ \& \ Gx) \supset \sim Ix) \ \checkmark \ a$ $\sim((Pa \ \& \ Ga) \supset \sim Ia) \ \checkmark$ $Pa \ \& \ Ga$ $\sim \sim Ia \ \checkmark$ Ia $(Pb \ \& \ Gb) \supset Ub \ \checkmark$ $\sim(Pb \ \& \ Gb) \quad Ub$	<p>B.*</p> $(\forall x)((Px \ \& \ Gx) \supset Ux) \ \backslash a$ $(\forall x)((Px \ \& \ Ux) \supset \sim Ix)$ $\sim(\forall x)((Px \ \& \ Gx) \supset \sim Ix) \ \checkmark \ a$ $\sim((Pa \ \& \ Ga) \supset \sim Ia) \ \checkmark$ $Pa \ \& \ Ga$ $\sim \sim Ia \ \checkmark$ Ia $(Pa \ \& \ Ga) \supset Ua \ \checkmark$ $\sim(Pa \ \& \ Ga) \quad Ua$ <p style="text-align: center;">X</p>
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A.*

$(\forall x)((Px \ \& \ Gx) \supset Ux) \ \backslash a$
 $(\forall x)((Px \ \& \ Ux) \supset \sim lx) \ \backslash a$
 $\sim(\forall x)((Px \ \& \ Gx) \supset \sim lx) \ \checkmark \ a$

$\sim((Pa \ \& \ Ga) \supset \sim la) \ \checkmark$

$Pa \ \& \ Ga$

$\sim \sim la \ \checkmark$

la

$(Pa \ \& \ Ga) \supset Ua \ \checkmark$

$\sim(Pa \ \& \ Ga)$
X

Ua

$(Pa \ \& \ Ua) \supset \sim la \ \checkmark$

$\sim(Pa \ \& \ Ua)$

$\sim la$
X

B.

$(\forall x)((Px \ \& \ Gx) \supset Ux) \ \backslash a$
 $(\forall x)((Px \ \& \ Ux) \supset \sim lx) \ \backslash a$
 $\sim(\forall x)((Px \ \& \ Gx) \supset \sim lx) \ \checkmark \ a$

$\sim((Pa \ \& \ Ga) \supset \sim la) \ \checkmark$

$Pa \ \& \ Ga$

$\sim \sim la \ \checkmark$

la

$(Pa \ \& \ Ga) \supset Ua \ \checkmark$

$\sim(Pa \ \& \ Ga)$
X

Ua

$(Pa \ \& \ Ua) \supset \sim la \ \checkmark$

$\sim(Pa \ \& \ Ua)$

$\sim \sim la \ \checkmark$

la

A.*

$$(\forall x)((Px \ \& \ Gx) \supset Ux) \ \backslash a$$

$$(\forall x)((Px \ \& \ Ux) \supset \sim lx) \ \backslash a$$

$$\sim(\forall x)((Px \ \& \ Gx) \supset \sim lx) \ \checkmark a$$

$$\sim((Pa \ \& \ Ga) \supset \sim la) \ \checkmark$$

$$Pa \ \& \ Ga$$

$$\sim \sim la \ \checkmark$$

$$la$$

$$(Pa \ \& \ Ga) \supset Ua \ \checkmark$$

$$\sim(Pa \ \& \ Ga)$$

X

$$Ua$$

$$(Pa \ \& \ Ua) \supset \sim la \ \checkmark$$

$$\sim(Pa \ \& \ Ua)$$

$$\sim la$$

X

$$\sim Pa$$

$$\sim Ua$$

X

B.

$$(\forall x)((Px \ \& \ Gx) \supset Ux) \ \backslash a$$

$$(\forall x)((Px \ \& \ Ux) \supset \sim lx) \ \backslash a$$

$$\sim(\forall x)((Px \ \& \ Gx) \supset \sim lx) \ \checkmark a$$

$$\sim((Pa \ \& \ Ga) \supset \sim la) \ \checkmark$$

$$Pa \ \& \ Ga$$

$$\sim \sim la \ \checkmark$$

$$la$$

$$(Pa \ \& \ Ga) \supset Ua \ \checkmark$$

$$\sim(Pa \ \& \ Ga)$$

X

$$Ua$$

$$(Pa \ \& \ Ua) \supset \sim la \ \checkmark$$

$$\sim(Pa \ \& \ Ua)$$

$$\sim la$$

X

$$\sim Pa$$

$$\sim Ua$$

X

A.

$$(\forall x)((Px \ \& \ Gx) \supset Ux) \ \backslash a$$

$$(\forall x)((Px \ \& \ Ux) \supset \sim lx) \ \backslash a$$

$$\sim(\forall x)((Px \ \& \ Gx) \supset \sim lx) \ \checkmark a$$

$$\sim((Pa \ \& \ Ga) \supset \sim la) \ \checkmark$$

$$Pa \ \& \ Ga \ \checkmark$$

$$\sim \sim la \ \checkmark$$

$$la$$

$$(Pa \ \& \ Ga) \supset Ua \ \checkmark$$

$$\sim(Pa \ \& \ Ga)$$

$$\mathbf{X}$$

$$Ua$$

$$(Pa \ \& \ Ua) \supset \sim la \ \checkmark$$

$$\sim(Pa \ \& \ Ua)$$

$$\sim la$$

$$\mathbf{X}$$

$$\sim Pa$$

$$\sim Ua$$

$$\mathbf{X}$$

$$Pa$$

$$Ga$$

$$\mathbf{X}$$

B.*

$$(\forall x)((Px \ \& \ Gx) \supset Ux) \ \backslash a$$

$$(\forall x)((Px \ \& \ Ux) \supset \sim lx) \ \backslash a$$

$$\sim(\forall x)((Px \ \& \ Gx) \supset \sim lx) \ \checkmark a$$

$$\sim((Pa \ \& \ Ga) \supset \sim la) \ \checkmark$$

$$Pa \ \& \ Ga \ \checkmark$$

$$\sim \sim la \ \checkmark$$

$$la$$

$$(Pa \ \& \ Ga) \supset Ua \ \checkmark$$

$$\sim(Pa \ \& \ Ga)$$

$$\mathbf{X}$$

$$Ua$$

$$(Pa \ \& \ Ua) \supset \sim la \ \checkmark$$

$$\sim(Pa \ \& \ Ua)$$

$$\sim la$$

$$\mathbf{X}$$

$$\sim Pa$$

$$\sim Ua$$

$$\mathbf{X}$$

$$Pa$$

$$Ga$$

$$\mathbf{X}$$

A person cannot be guilty of murder unless they are capable of understanding that their action caused the death of another person. But no one who is capable of understanding that their action caused the death of another person is insane. Consequently, no one who is guilty of murder is insane.

1. $(\forall x)((Px \ \& \ Gx) \supset Ux)$

2. $(\forall x)((Px \ \& \ Ux) \supset \sim Ix)$

Therefore:

C. $(\forall x)((Px \ \& \ Gx) \supset \sim Ix)$

Given that all the branches for the tree are closed, which of the following statements about the argument is correct?

A.* The argument is valid.

B. The argument is invalid.

C. We cannot yet tell whether the argument is valid or invalid.

Is the following argument form valid?

$$(\forall x)(\sim Lxx) \vdash (\exists x)(\forall y)(Lxy \supset Lyy)$$

How does the tree to test this argument begin?

A.* $(\forall x)(\sim Lxx)$ $\sim(\exists x)(\forall y)(Lxy \supset Lyy)$	B. $(\forall x)(\sim Lxx)$ $(\exists x)(\forall y)(Lxy \supset Lyy)$
C. $\sim(\forall x)(\sim Lxx)$ $\sim(\exists x)(\forall y)(Lxy \supset Lyy)$	D. $\sim(\forall x)(\sim Lxx)$ $(\exists x)(\forall y)(Lxy \supset Lyy)$

How should the tree continue?

<p>A.</p> $ \begin{array}{c} (\forall x)(\sim Lxx) \setminus a \\ \sim(\exists x)(\forall y)(Lxy \supset Lyy) \setminus a \\ \quad \mid \\ \quad \sim Laa \\ \quad \quad \mid \\ \quad \quad \sim(\forall y)(Lay \supset Lyy) \checkmark a \\ \quad \quad \quad \mid \\ \quad \quad \quad \sim(Laa \supset Laa) \end{array} $	<p>B.*</p> $ \begin{array}{c} (\forall x)(\sim Lxx) \setminus a \\ \sim(\exists x)(\forall y)(Lxy \supset Lyy) \setminus a \\ \quad \mid \\ \quad \sim Laa \\ \quad \quad \mid \\ \quad \quad \sim(\forall y)(Lay \supset Lyy) \checkmark b \\ \quad \quad \quad \mid \\ \quad \quad \quad \sim(Lab \supset Lbb) \end{array} $
<p>C.</p> $ \begin{array}{c} (\forall x)(\sim Lxx) \setminus a \\ \sim(\exists x)(\forall y)(Lxy \supset Lyy) \setminus a \\ \quad \mid \\ \quad \sim Laa \\ \quad \quad \mid \\ \quad \quad \sim(\forall y)(Lay \supset Lyy) \checkmark b \\ \quad \quad \quad \mid \\ \quad \quad \quad (Lab \supset Lbb) \end{array} $	<p>D.</p> $ \begin{array}{c} (\forall x)(\sim Lxx) \setminus a \\ \sim(\exists x)(\forall y)(Lxy \supset Lyy) \setminus a \\ \quad \mid \\ \quad \sim Laa \\ \quad \quad \mid \\ \quad \quad \sim(\forall y)(Lay \supset Lyy) \checkmark b \\ \quad \quad \quad \swarrow \quad \searrow \\ \quad \quad \quad \sim Lab \quad \sim Lbb \end{array} $

How should the tree continue?

<p>A.</p> $ \begin{array}{c} (\forall x)(\sim Lxx) \setminus a \\ \sim(\exists x)(\forall y)(Lxy \supset Lyy) \setminus a \\ \quad \mid \\ \quad \sim Laa \\ \quad \quad \mid \\ \quad \quad \sim(\forall y)(Lay \supset Lyy) \checkmark b \\ \quad \quad \quad \mid \\ \quad \quad \quad \sim(Lab \supset Lbb) \checkmark \\ \quad \quad \quad \swarrow \quad \searrow \\ \quad \quad \quad \sim Lab \quad \sim Lbb \end{array} $	<p>B.</p> $ \begin{array}{c} (\forall x)(\sim Lxx) \setminus a \\ \sim(\exists x)(\forall y)(Lxy \supset Lyy) \setminus a \\ \quad \mid \\ \quad \sim Laa \\ \quad \quad \mid \\ \quad \quad \sim(\forall y)(Lay \supset Lyy) \checkmark b \\ \quad \quad \quad \mid \\ \quad \quad \quad \sim(Lab \supset Lbb) \checkmark \\ \quad \quad \quad \swarrow \quad \searrow \\ \quad \quad \quad \sim Lab \quad Lbb \end{array} $
<p>C.</p> $ \begin{array}{c} (\forall x)(\sim Lxx) \setminus a \\ \sim(\exists x)(\forall y)(Lxy \supset Lyy) \setminus a \\ \quad \mid \\ \quad \sim Laa \\ \quad \quad \mid \\ \quad \quad \sim(\forall y)(Lay \supset Lyy) \checkmark b \\ \quad \quad \quad \mid \\ \quad \quad \quad \sim(Lab \supset Lbb) \checkmark \\ \quad \quad \quad \mid \\ \quad \quad \quad \sim Lab \\ \quad \quad \quad Lbb \end{array} $	<p>D.*</p> $ \begin{array}{c} (\forall x)(\sim Lxx) \setminus a \\ \sim(\exists x)(\forall y)(Lxy \supset Lyy) \setminus a \\ \quad \mid \\ \quad \sim Laa \\ \quad \quad \mid \\ \quad \quad \sim(\forall y)(Lay \supset Lyy) \checkmark b \\ \quad \quad \quad \mid \\ \quad \quad \quad \sim(Lab \supset Lbb) \checkmark \\ \quad \quad \quad \mid \\ \quad \quad \quad Lab \\ \quad \quad \quad \sim Lbb \end{array} $

How should the tree continue?

<p>A.*</p> $\begin{array}{c} (\forall x)(\sim Lxx) \setminus a \setminus b \\ \sim(\exists x)(\forall y)(Lxy \supset Lyy) \setminus a \setminus b \\ \quad \\ \quad \sim Laa \\ \quad \quad \\ \quad \quad \sim(\forall y)(Lay \supset Lyy) \checkmark b \\ \quad \quad \quad \\ \quad \quad \quad \sim(Lab \supset Lbb) \checkmark \\ \quad \quad \quad \quad \\ \quad \quad \quad \quad Lab \\ \quad \quad \quad \quad \quad \\ \quad \quad \quad \quad \quad \sim Lbb \\ \quad \quad \quad \quad \quad \quad \\ \quad \quad \quad \quad \quad \quad \sim Lbb \\ \quad \quad \quad \quad \quad \quad \quad \\ \quad \quad \quad \quad \quad \quad \quad \sim(\forall y)(Lby \supset Lyy) \checkmark c \\ \quad \quad \quad \quad \quad \quad \quad \quad \\ \quad \quad \quad \quad \quad \quad \quad \quad \sim(Lbc \supset Lcc) \checkmark \\ \quad \quad \quad \quad \quad \quad \quad \quad \quad \\ \quad \quad \quad \quad \quad \quad \quad \quad \quad Lbc \\ \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \\ \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \sim Lcc \end{array}$	<p>B.</p> $\begin{array}{c} (\forall x)(\sim Lxx) \setminus a \setminus b \\ \sim(\exists x)(\forall y)(Lxy \supset Lyy) \setminus a \setminus b \\ \quad \\ \quad \sim Laa \\ \quad \quad \\ \quad \quad \sim(\forall y)(Lay \supset Lyy) \checkmark b \\ \quad \quad \quad \\ \quad \quad \quad \sim(Lab \supset Lbb) \checkmark \\ \quad \quad \quad \quad \\ \quad \quad \quad \quad Lab \\ \quad \quad \quad \quad \quad \\ \quad \quad \quad \quad \quad \sim Lbb \\ \quad \quad \quad \quad \quad \quad \\ \quad \quad \quad \quad \quad \quad \sim Lbb \\ \quad \quad \quad \quad \quad \quad \quad \\ \quad \quad \quad \quad \quad \quad \quad \sim(\forall y)(Lby \supset Lyy) \setminus b \\ \quad \quad \quad \quad \quad \quad \quad \quad \\ \quad \quad \quad \quad \quad \quad \quad \quad \sim(Lbb \supset Lbb) \checkmark \\ \quad \quad \quad \quad \quad \quad \quad \quad \quad \\ \quad \quad \quad \quad \quad \quad \quad \quad \quad Lbb \\ \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \\ \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \sim Lbb \\ \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \\ \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad X \end{array}$
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$$\begin{array}{c}
 (\forall x)(\sim Lxx) \setminus a \setminus b \\
 \sim(\exists x)(\forall y)(Lxy \supset Lyy) \setminus a \setminus b \\
 | \\
 \sim Laa \\
 | \\
 \sim(\forall y)(Lay \supset Lyy) \checkmark b \\
 | \\
 \sim(Lab \supset Lbb) \checkmark \\
 | \\
 Lab \\
 | \\
 \sim Lbb \\
 | \\
 \sim Lbb \\
 | \\
 \sim(\forall y)(Lby \supset Lyy) \checkmark c \\
 | \\
 \sim(Lbc \supset Lcc) \checkmark \\
 | \\
 Lbc \\
 | \\
 \sim Lcc \\
 | \\
 \dots
 \end{array}$$

Do you think this tree will continue indefinitely, without ever closing?

- A. No, the tree eventually closes.
- B.* Yes, the tree will continue indefinitely, without ever closing.
- C. It's impossible to tell.
- D. Don't know.

$$\begin{array}{c}
 (\forall x)(\sim Lxx) \setminus a \setminus b \\
 \sim(\exists x)(\forall y)(Lxy \supset Lyy) \setminus a \setminus b \\
 | \\
 \sim Laa \\
 | \\
 \sim(\forall y)(Lay \supset Lyy) \checkmark b \\
 | \\
 \sim(Lab \supset Lbb) \checkmark \\
 | \\
 Lab \\
 \sim Lbb \\
 | \\
 \sim Lbb \\
 | \\
 \sim(\forall y)(Lby \supset Lyy) \checkmark c \\
 | \\
 \sim(Lbc \supset Lcc) \checkmark \\
 | \\
 Lbc \\
 \sim Lcc \\
 | \\
 \dots
 \end{array}$$

What does this imply about the validity of the argument?

- A. It is valid.
- B.* It is invalid.
- C. It's impossible to tell.
- D. Don't know.

$$\begin{array}{c}
 (\forall x)(\sim Lxx) \setminus a \setminus b \\
 \sim(\exists x)(\forall y)(Lxy \supset Lyy) \setminus a \setminus b \\
 | \\
 \sim Laa \\
 | \\
 \sim(\forall y)(Lay \supset Lyy) \checkmark b \\
 | \\
 \sim(Lab \supset Lbb) \checkmark \\
 | \\
 Lab \\
 | \\
 \sim Lbb \\
 | \\
 \sim Lbb \\
 | \\
 \sim(\forall y)(Lby \supset Lyy) \checkmark c \\
 | \\
 \sim(Lbc \supset Lcc) \checkmark \\
 | \\
 Lbc \\
 | \\
 \sim Lcc \\
 | \\
 \dots
 \end{array}$$

A completed branch for this tree will contain the formulas:

- A. $\sim Laa, Lab, \sim Lbb, \sim Lab, Lbc, \sim Lcc, \sim Lbc, Lcd, \sim Ldd \dots$
- B. $\sim Laa, Lab, \sim Lbb, Lba, Lbc, \sim Lcc, Lcb, Lbc, \sim Ldd, Lde, Led, \sim Lee \dots$
- C.* $\sim Laa, Lab, \sim Lbb, Lbc, \sim Lcc, Lcd, \sim Ldd, Lde, \sim Lee \dots$
- D. $\sim Laa, Lab, \sim Lbb, Lac, \sim Lcc, Lad, \sim Ldd, Lae, \sim Lee \dots$

A completed branch for this tree will contain the formulas:

$\sim L_{aa}$, L_{ab} , $\sim L_{bb}$, L_{bc} , $\sim L_{cc}$, L_{cd} , $\sim L_{dd}$, L_{de} , $\sim L_{ee}$...

Which of the following could be true?

- I. $a = b$
 - II. $b = c$
 - III. $a = c$
 - IV. $c = d$
- A. I or II
- B.* III only
- C. IV only
- D. None of the above

A completed branch for this tree will contain the formulas:

$\sim L_{aa}, L_{ab}, \sim L_{bb}, L_{bc}, \sim L_{cc}, L_{cd}, \sim L_{dd}, L_{de}, \sim L_{ee} \dots$

What is size of the *smallest* domain that invalidates the argument?

- A. One object
- B.* Two objects
- C. Three objects
- D. Four objects

Consider the following argument:

All cats are mammals. Therefore, anyone who owns a cat owns a mammal.

Traditional Aristotelean logic was unable to account for the validity of this form of argument. Can we do better? We will translate the argument into the language of the predicate calculus then test for validity using the tree method.

1. All cats are mammals

Cx - x is a cat

Mx - x is a mammal

A. $(\exists x)(Cx \supset Mx)$

B.* $(\forall x)(Cx \supset Mx)$

C. $(\forall x)(Mx \supset Cx)$

D.* $(\forall x)(\sim Mx \supset \sim Cx)$

Consider the following argument:

All cats are mammals. Therefore, anyone who owns a cat owns a mammal.

Traditional Aristotelean logic was unable to account for the validity of this form of argument. Can we do better? We will translate the argument into the language of the predicate calculus then test for validity using the tree method.

2. Any cat owner owns a mammal

Cx - x is a cat

Mx - x is a mammal

Oxy - x owns y

A. $(\forall x)((\exists y)(Cy \ \& \ Oyx) \supset (\exists z)(Mz \ \& \ Ozx))$

B. $(\forall x)((\exists y)(Cy \ \& \ Oxy) \ \& \ (\exists z)(Mz \ \& \ Ozx))$

C. $(\forall x)(\exists y)((Cy \ \& \ Oxy) \supset (My \ \& \ Oxy))$

D.* $(\forall x)((\exists y)(Cy \ \& \ Oxy) \supset (\exists z)(Mz \ \& \ Ozx))$

$$1. (\forall x)(Cx \supset Mx)$$

Therefore:

$$C. (\forall x)((\exists y)(Cy \ \& \ Oxy) \supset (\exists z)(Mz \ \& \ Oxz))$$

How does the tree to test this argument begin?

A.	$(\forall x)(Cx \supset Mx)$ $(\forall x)((\exists y)(Cy \ \& \ Oxy) \supset (\exists z)(Mz \ \& \ Oxz))$
B.*	$(\forall x)(Cx \supset Mx)$ $\sim(\forall x)((\exists y)(Cy \ \& \ Oxy) \supset (\exists z)(Mz \ \& \ Oxz))$
C.	$\sim(\forall x)(Cx \supset Mx)$ $\sim(\forall x)((\exists y)(Cy \ \& \ Oxy) \supset (\exists z)(Mz \ \& \ Oxz))$
D.	$(\forall x)(Cx \supset Mx)$ $(\forall x)\sim((\exists y)(Cy \ \& \ Oxy) \supset (\exists z)(Mz \ \& \ Oxz))$

How should the tree continue?

A.

$$\begin{array}{c} (\forall x)(Cx \supset Mx) \\ \sim(\forall x)((\exists y)(Cy \ \& \ Oxy) \supset (\exists z)(Mz \ \& \ Oxz)) \ \checkmark a \\ | \\ \sim((\exists y)(Cy \ \& \ Oay) \supset (\exists z)(Mz \ \& \ Oaz)) \ \checkmark \\ / \quad \backslash \\ \sim(\exists y)(Cy \ \& \ Oay) \quad (\exists z)(Mz \ \& \ Oaz) \end{array}$$

B.*

$$\begin{array}{c} (\forall x)(Cx \supset Mx) \\ \sim(\forall x)((\exists y)(Cy \ \& \ Oxy) \supset (\exists z)(Mz \ \& \ Oxz)) \ \checkmark a \\ | \\ \sim((\exists y)(Cy \ \& \ Oay) \supset (\exists z)(Mz \ \& \ Oaz)) \ \checkmark \\ | \\ (\exists y)(Cy \ \& \ Oay) \\ \sim(\exists z)(Mz \ \& \ Oaz) \end{array}$$

C.

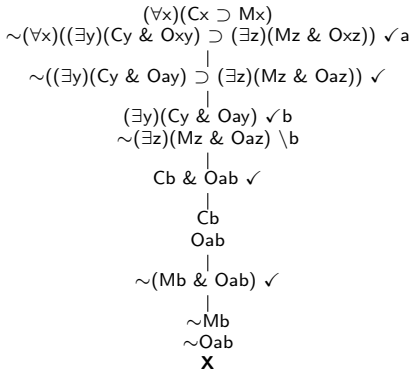
$$\begin{array}{c} (\forall x)(Cx \supset Mx) \\ \sim(\forall x)((\exists y)(Cy \ \& \ Oxy) \supset (\exists z)(Mz \ \& \ Oxz)) \ \checkmark a \\ | \\ \sim((\exists y)(Cy \ \& \ Oay) \supset (\exists z)(Mz \ \& \ Oaz)) \ \backslash b \\ | \\ \sim((Cb \ \& \ Oab) \supset (\exists z)(Mz \ \& \ Oaz)) \end{array}$$

How should the tree continue?

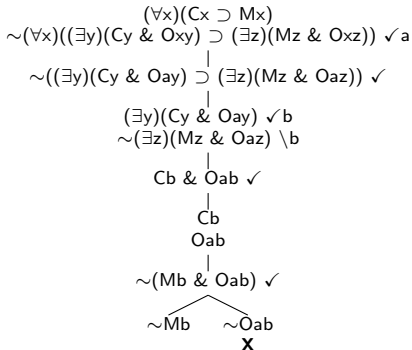
<p>A.*</p> $ \begin{array}{c} (\forall x)(Cx \supset Mx) \\ \sim(\forall x)((\exists y)(Cy \ \& \ Oxy) \supset (\exists z)(Mz \ \& \ Oxz)) \ \checkmark a \\ \sim((\exists y)(Cy \ \& \ Oay) \supset (\exists z)(Mz \ \& \ Oaz)) \ \checkmark \\ (\exists y)(Cy \ \& \ Oay) \ \checkmark b \\ \sim(\exists z)(Mz \ \& \ Oaz) \\ Cb \ \& \ Oab \ \checkmark \\ Cb \\ Oab \end{array} $	<p>B.</p> $ \begin{array}{c} (\forall x)(Cx \supset Mx) \\ \sim(\forall x)((\exists y)(Cy \ \& \ Oxy) \supset (\exists z)(Mz \ \& \ Oxz)) \ \checkmark a \\ \sim((\exists y)(Cy \ \& \ Oay) \supset (\exists z)(Mz \ \& \ Oaz)) \ \checkmark \\ (\exists y)(Cy \ \& \ Oay) \ \checkmark a \\ \sim(\exists z)(Mz \ \& \ Oaz) \\ Ca \ \& \ Oaa \ \checkmark \\ Ca \\ Oaa \end{array} $
<p>C.</p> $ \begin{array}{c} (\forall x)(Cx \supset Mx) \\ \sim(\forall x)((\exists y)(Cy \ \& \ Oxy) \supset (\exists z)(Mz \ \& \ Oxz)) \ \checkmark a \\ \sim((\exists y)(Cy \ \& \ Oay) \supset (\exists z)(Mz \ \& \ Oaz)) \ \checkmark \\ (\exists y)(Cy \ \& \ Oay) \ \checkmark b \\ \sim(\exists z)(Mz \ \& \ Oaz) \\ \sim(Cb \ \& \ Oab) \ \checkmark \\ \sim Cb \\ \sim Oab \end{array} $	<p>D.</p> $ \begin{array}{c} (\forall x)(Cx \supset Mx) \\ \sim(\forall x)((\exists y)(Cy \ \& \ Oxy) \supset (\exists z)(Mz \ \& \ Oxz)) \ \checkmark a \\ \sim((\exists y)(Cy \ \& \ Oay) \supset (\exists z)(Mz \ \& \ Oaz)) \ \checkmark \\ (\exists y)(Cy \ \& \ Oay) \ \checkmark b \\ \sim(\exists z)(Mz \ \& \ Oaz) \\ \sim(Cb \ \& \ Oab) \ \checkmark \\ \sim Cb \quad \sim Oab \end{array} $

How should the tree continue?

A.



B.*



All cats are mammals. Therefore, anyone who owns a cat owns a mammal.

$$1. (\forall x)(Cx \supset Mx)$$

Therefore:

$$C. (\forall x)((\exists y)(Cy \ \& \ Oxy) \supset (\exists z)(Mz \ \& \ Oxz))$$

The tree for this argument form shows that the argument is:

A.* Valid.

B. Invalid.

C. Impossible to decide.

Here is one of Lewis Carroll's logic puzzles:

1. When I work a logic-example without grumbling, you may be sure it is one that I can understand.
2. Examples invented by Lewis Carroll are not arranged in regular order, like the examples I am used to.
3. No easy example ever makes my head ache.
4. I can't understand examples that are not arranged in regular order, like the examples I am used to.
5. I never grumble at an example, unless it gives me a headache.

The puzzle is to work out the conclusion that follows from all five premises. We will first translate the premises into symbols.

5. I never grumble at an example, unless it gives me a headache.

Gx - Logic examples that make me grumble.

Hx - Logic examples that make my head ache.

A.* $(\forall x)(Gx \supset Hx)$

B. $(\forall x)(Hx \supset Gx)$

C. $(\forall x)(Gx \supset \sim Hx)$

D. $(\forall x)(\sim Hx \supset Gx)$

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The puzzle is to work out the conclusion that follows from all five premises. We will first translate the premises into symbols.

4. I can't understand examples that are not arranged in regular order, like the examples I am used to.

Ox - Logic examples arranged in regular order, like the examples I am used to.

Ux - Logic examples that are understood by me.

A.* $(\forall x)(\sim O_x \supset \sim U_x)$

B. $(\forall x)(O_x \supset U_x)$

C. $(\exists x)(\sim O_x \ \& \ \sim U_x)$

D. $\sim(\forall x)(O_x \supset U_x)$

Here is one of Lewis Carroll's logic puzzles:

1. When I work a logic-example without grumbling, you may be sure it is one that I can understand.
2. Examples invented by Lewis Carroll are not arranged in regular order, like the examples I am used to.
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5. I never grumble at an example, unless it gives me a headache.

The puzzle is to work out the conclusion that follows from all five premises. We will first translate the premises into symbols.

3. No easy example ever makes my head ache.

Ex - Logic examples that are easy.

Hx - Logic examples that make my head ache.

A. $(\exists x)\sim(Ex \ \& \ Hx)$

B. $\sim(\forall x)(Ex \ \& \ \sim Hx)$

C. $\sim(\exists x)(Ex \ \& \ \sim Hx)$

D.* $(\forall x)(Ex \ \supset \ \sim Hx)$

Here is one of Lewis Carroll's logic puzzles:

1. When I work a logic-example without grumbling, you may be sure it is one that I can understand.
2. Examples invented by Lewis Carroll are not arranged in regular order, like the examples I am used to.
3. No easy example ever makes my head ache.
4. I can't understand examples that are not arranged in regular order, like the examples I am used to.
5. I never grumble at an example, unless it gives me a headache.

The puzzle is to work out the conclusion that follows from all five premises. We will first translate the premises into symbols.

2. Examples invented by Lewis Carroll are not arranged in regular order, like the examples I am used to.

Ox - Logic examples arranged in regular order, like the examples I am used to.

Cx - Logic examples invented by Lewis Carroll.

A. $(\forall x)(Cx \supset Ox)$

B.* $(\forall x)(Cx \supset \sim Ox)$

C. $(\forall x)(\sim Cx \supset \sim Ox)$

D. $(\exists x)(Cx \ \& \ \sim Ox)$

Here is one of Lewis Carroll's logic puzzles:

1. When I work a logic-example without grumbling, you may be sure it is one that I can understand.
2. Examples invented by Lewis Carroll are not arranged in regular order, like the examples I am used to.
3. No easy example ever makes my head ache.
4. I can't understand examples that are not arranged in regular order, like the examples I am used to.
5. I never grumble at an example, unless it gives me a headache.

The puzzle is to work out the conclusion that follows from all five premises. We will first translate the premises into symbols.

1. When I work a logic-example without grumbling, you may be sure it is one that I can understand.

Gx - Logic examples that make me grumble.

Ux - Logic examples that are understood by me.

A. $(\exists x)(\sim Gx \ \& \ Ux)$

B. $(\forall x)(\sim Ux \supset \sim Gx)$

C.* $(\forall x)(\sim Gx \supset Ux)$

D. $(\forall x)(Ux \supset \sim Gx)$

Here is one of Lewis Carroll's logic puzzles:

1. $(\forall x)(\sim Gx \supset Ux)$
2. $(\forall x)(Cx \supset \sim O_x)$
3. $(\forall x)(Ex \supset \sim Hx)$
4. $(\forall x)(\sim O_x \supset \sim Ux)$
5. $(\forall x)(Gx \supset Hx)$

<p>Gx - Logic examples that make me grumble. Ux - Logic examples that are understood by me. Ox - Logic examples arranged in regular order ... Cx - Logic examples invented by Lewis Carroll. Hx - Logic examples that make my head ache. Ex - Logic examples that are easy.</p>

A student claimed that the conclusion that follows from all five premises is:

Logic examples invented by Lewis Carroll make my made head ache.

Construct a tree to work out if the student was correct. (Work in pairs)

- A. The student's answer does follow from all five premise.
- B.* The student's answer does not follow from all five premises.

NOTE: The given statement follows from the premises, but not from ALL of them. Premise 3 is not used.

Here is one of Lewis Carroll's logic puzzles:

1. $(\forall x)(\sim Gx \supset Ux)$
2. $(\forall x)(Cx \supset \sim Ox)$
3. $(\forall x)(Ex \supset \sim Hx)$
4. $(\forall x)(\sim Ox \supset \sim Ux)$
5. $(\forall x)(Gx \supset Hx)$

<p>Gx - Logic examples that make me grumble. Ux - Logic examples that are understood by me. Ox - Logic examples arranged in regular order ... Cx - Logic examples invented by Lewis Carroll. Hx - Logic examples that make my head ache. Ex - Logic examples that are easy.</p>

Can you work out the conclusion that *does* follow from all five premises?

- A. The only logic examples that make my head ache are invented by Lewis Carroll.
- B. No easy example makes my head ache.
- C. Every logic example invented by Lewis Carroll is easy.
- D.* No logic example invented by Lewis Carroll is easy.